

Relational Algebra: The basic set of operations for the relational model is the relational algebra. These operations enable a user to specify basic retrieval requests as relational algebra expressions.

The relational algebra provides a formal foundation for relational model operations. It is also used as a basis for implementing and optimizing queries in the query processing and optimization modules that are integral parts of RDBMSs.

operations of Relational Algebra:-

Select:- selects all tuples that satisfy the selection condition from a relation R.

Notation:- $\sigma_{\langle \text{selection condition} \rangle} (R)$

Eg:- select the tuples for all employees who either work in dept 4 and make over 25,000 per year, or work in dept 5 and make over 35000.

$\sigma_{(Dno=4 \text{ and } Salary > 25000) \text{ or } (Dno=5 \text{ and } Salary > 35000)} (Emp)$

Project:- produces ~~all tuples that satisfy~~ a new relation with only some of the attributes of R, and removes duplicate tuples.

Notation:- $\pi_{\langle \text{attribute list} \rangle} (R)$

eg:- List each employee's first and last name and salary.

$\pi_{Fname, Lname, Salary} (Emp)$

Rename :- Relation names and attribute names can be renamed using ρ operator.

notation :- $\rho_S(R)$ or $\rho_{S(A_1, A_2; B_1, B_2)}(R)$ or $\rho_{(A_1, A_2; B_1, B_2)}(R)$

eg :- $\rho_{emp}(employee)$

Relational algebra operations from Set Theory :-

UNION, INTERSECT, MINUS (SET DIFFERENCE)

UNION :- produces a relation that includes all the tuples in ~~both~~ R_1 ^{or} ~~and~~ R_2 or both R_1 and R_2 ; R_1 and R_2 must be union compatible - Same no. of attributes and each pair of attributes from same domain.

notation :- $R_1 \cup R_2$

eg :- Retrieve the SSNs of all employees who either work in dept 5 or directly supervise an employee who works in dept 5.

$depts_emps \leftarrow \sigma_{dno=5}(Emp)$
 $Result1 \leftarrow \pi_{ssn}(depts_emps)$
 $Result2(ssn) \leftarrow \pi_{super_ssn}(depts_emps)$
 $Result \leftarrow Result1 \cup Result2$

Intersection :- produces a relation that includes all the tuples in both R_1 and R_2 ; R_1 and R_2 must be union compatible.

notation :- $R_1 \cap R_2$

eg :- $student \cap instructor$

Difference :- produces a relation that includes all the tuples in R_1 that are not in R_2 ; R_1 and R_2 must be union compatible.

notation :- $R_1 - R_2$

eg :- $student - instructor$

Cartesian product :- produces a relation that has the attributes of R_1 and R_2 and includes all tuples all combinations of tuples from R_1 and R_2
notation :- $R_1 \times R_2$

Binary relational operations JOIN and Division: -

The JOIN operation denoted by \bowtie , is used to combine related tuples from two relations into single "longer" tuples.

notation: - $R \bowtie_{\langle \text{join-condition} \rangle} S$

The result of the join is a relation Q with n+m attributes $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$ in that order; Q has one tuple for each combination of tuples - one from R and one from S - whenever the combination satisfies the join condition.

Eg: - Retrieve the name of the manager of each department

$\text{Dept-mgr} \leftarrow \text{Dept} \bowtie_{\text{mgr-ssn} = \text{ssn}} \text{Emp}$

$\text{Result} \leftarrow \Pi_{\text{Dname, Lname, Fname}} (\text{Dept-mgr})$

The main difference between Cartesian product and join is:

In join, only combinations of tuples satisfying the join condition appear in the result, whereas in the Cartesian product all combinations of tuples are included in the result.

A join operation with such a general join condition is called a Theta Join (θ) ($\theta = \{=, <, \leq, >, \geq, \neq\}$)

A join, where the only comparison operator used is =, is called an Equi join.

A natural join denoted by $*$ was created to get rid of the second attribute in an "Equi join condition".

Eg: - $\text{proj-dept} \leftarrow \text{Project} * \text{Dept}$

For natural join the two join attributes have to be same name in both relations. If not, a renaming operation is applied first

Eg: - $\text{Proj-dept} \leftarrow \text{Project} * \rho_{(\text{Dname, Dnum, mgr-ssn})} (\text{Department})$
The attribute Dnum is the join attribute here.

A Complete set of Relational Algebra operations :-

The set of relational algebra operations $\{\sigma, \pi, \cup, \rho, -, \times\}$ is a complete set; that is, any of the other relational algebra operations can be expressed as a sequence of operations from this set.

Eg: - $R \cap S = (R \cup S) - ((R - S) \cup (S - R))$

~~R~~ $R \bowtie_{\langle \text{condition} \rangle} S = \sigma_{\text{Condition}}(R \times S)$

The Division operation :- The division operation is applied to two relations $R(Z) \div S(X)$, where the attributes of R are a subset of the attributes of S ; that is, $X \subseteq Z$.

Eg: - Retrieve the names of employees who work on all the projects that 'John Smith' works on.

- SMITH $\leftarrow \sigma_{\text{Fname}='John' \text{ and } \text{Lname}='Smith'}(\text{EMP})$
- SMITH_PNOs $\leftarrow \pi_{\text{PNO}}(\text{WORKS-ON} \bowtie_{\text{ESSN}=\text{SSN}} \text{SMITH})$
- SSN_PNOs $\leftarrow \pi_{\text{ESSN}, \text{PNO}}(\text{WORKS-ON})$
- SSNS $\leftarrow \text{SSN_PNOs} \div \text{SMITH_PNOs}$
- RESULT $\leftarrow \pi_{\text{Fname}, \text{Lname}}(\text{SSNS} \times \text{EMP})$

SSN_PNOs

ESSN	PNO
123456789	1
123456789	2
666888444	3
453453453	1
453453453	2
234455555	2
332445555	3
333445555	10
9989777	30
998777	10
9879578	30
9876543	20
888666555	20

SMITH_PNOs

PNO
1
2

SSNS

SSN
123456789
453453453

Let Y be the set of attributes of R that are not attributes of S , i.e. $Y = Z - X$

The result of division is a relation $T(Y)$ that includes a tuple t if tuples t_R appear in R with $t_R[Y] = t$ and with $t_R[X] = t_S$ for every tuple t_S in S .

Eg: - $X = \{A\}$ $Y = \{B\}$ and $Z = \{A, B\}$

R

A	B
a ₁	b ₁
a ₂	b ₁
a ₃	b ₁
a ₄	b ₁
a ₁	b ₂
a ₃	b ₂
a ₂	b ₃
a ₃	b ₃
a ₄	b ₃

S

A
a ₁
a ₂
a ₃

$R \div S \rightarrow T$

T

B
b ₁
b ₂

For a tuple t to appear in the result T of the division, the values in t must appear in R in combination with every tuple in S .

The division operation can be expressed as a sequence of π, \times and $-$ operations as follows:

$T_1 \leftarrow \pi_Y(R), T_2 \leftarrow \pi_Y((S \times T_1) - R)$
 $T \leftarrow T_1 - T_2$