Types of Recurrence Relations:

Two Types of Recurrence Relation.

① Linear Recurrence relation / Homogeneous Recurrence relation.

② non linear Recurrence relation / non homogeneous or Inhomogeneous recurrence relations

Note:- if the function $f(n)=0$ then the recurrence relation is said to homogeneous otherwise it is said to be non-homogeneous.

Order of the recurrence relation:

It is the difference b/w highest and lowest subscripts.

Example 1. $a_n + a_{n-1} = 0$.

$n-(n-1) = n-n+1 = 1^{st}$ order.

RR:

2. $a_n + a_{n-1} + a_{n-2} = 0$.

$n-(n-2) \Rightarrow n-n+2 = 2$ 2^{nd} order.

3. $a_n + a_{n-1} + a_{n-2} + \cdots a_{n-n} = 0$.

$n-(n-n) = n-n+n = n^{th}$ order!

4. $a_n + 2a_{n-1} = r^{r}$ — non homogeneous Recurrence relation.

$n-(n-1) = n-n+1 = 1^{st}$ order.

$a_n + 3a_{n-2} = 0$.

$n-(n-2) = n-n+2 = 2^{nd}$ order.

Example:

Recurrence Relation

- Iteration method
- Characteristic method.
- generating method.

Methods of solving Recurrence Relation

Using characteristic method

Procedure:-

1. write characteristic equation for the given recurrence relation.
2. Find the roots. and let roots are to be $r_1, r_2, r_3 \dots r_n$.
3. If r_1 and r_2 are real and distinct, then the solution is $\boxed{a_n = \alpha_1 r_1^n + \alpha_2 r_2^n}$ where α_1, α_2 are arbitrary constants.
4. If r_1 and r_2 are real and equal. then the solution is $\boxed{a_n = (\alpha_1 + \alpha_2 n) r^n}$ where α_1 & α_2 are arbitrary constants.
5. r_1 & r_2 are complex, then the solution is.

   $a_n = r^n(\alpha_1 \cos n\phi + \alpha_2 \cos n\phi)$. where α_1 & α_2 are arbitrary constants.

Relations:

Example: Find the solution of the recurrence relation

$a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n = 3, 4, 5,$ with

$a_0 = 3,\ a_1 = 6$ and $a_2 = 0$.

Sol: The Given Recurrence Relation is

$$a_n - 2a_{n-1} - a_{n-2} + 2a_{n-3} = 0.$$

The characteristic equation is

$$r^3 - 2r^2 - r + 2 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & -2 & -1 & 2 \\ & 0 & 1 & -1 & -2 \\ \hline & 1 & -1 & -2 & \underline{|0} \end{array}$$

$(r-1)(r^2 - r - 2) = 0.$

$(r-1)(r+1)(r-2) = 0.$

$\Rightarrow r = 1, 2, -1.$

$c_1 + c_2 + c_3 = 3$

$\cancel{c_1} + 2c_2 - c_3 = 6$

$3c_2 = 9$

$c_2 = 9/3 = 3$

Hence the solution is.

$$a_n = c_1 1^n + c_2 2^n + c_3(-1)^n.$$

where c_1, c_2, c_3 are arbitrary constants.

Initial conditions are:

$a_0 = 3,\ a_1 = 6$ and $a_2 = 0$.

if $a_0 = 3$, $a_0 = c_1 1^0 + c_2 2^0 + c_3(-1)^0$

$\Rightarrow c_1 + c_2 + c_3 = 3. \Rightarrow c_1 + c_2 + c_3 = 3$ — ①

when $a_1 = 6$, $a_1 = c_1 1^1 + c_2 2^1 + c_3(-1)^1$

$c_1 + 2c_2 - c_3 = 6.$ — ②

If $a_2 = 0$:

$$c_1 + c_2 2^2 + c_3(-1)^2 = 0$$

$$\Rightarrow 4c_2 + c_3 = 0 \text{ ——— } (3)$$

$c_1 = 6$

$c_2 = -1$

$c_3 = -2$

$a_0 = 3$
$a_1 = 6$
$a_2 = 0$

$6 - 1 - 2 \ldots$
$6 - 4 - 2$

The unique solution is

$$\boxed{a_n = 6(1)^n - 2^n - 2(-1)^n}$$

Example: Solve the recurrence relation ~~[illegible]~~

$a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 2$

given that $a_0 = -1$ and $a_1 = 8$

Solution: Given Recurrence relation is

$$a_n + a_{n-1} - 6a_{n-2} = 0$$

characteristic equation of given recurrence relation.

$$r^2 + r - 6 = 0$$

$$r(r+3) - 2(r+3) = 0$$

$$(r-2)(r+3) = 0$$

$$\therefore r = 2, -3$$

roots are real and distinct

Case 1: The general solution is.

$$a_n = c_1(2)^n + c_2(-3)^n \qquad \therefore a_n = A\cdot(r_1)^n + B\cdot(r_2)^n$$

c_1 & c_2 are constants.

The initial conditions are

$a_0 = -1$, $a_1 = 8$

Now take $a_0 = -1$ i.e $n = 0$.

$a_0 = c_1 \cdot (2)^0 + c_2 (-3)^0$

$c_1 + c_2 = -1$ —— (1)

$a_1 = c_1 \cdot 2^1 + c_2 (-3)^1$

$2c_1 - 3c_2 = 8$ —— (2)

from (1) & (2)

$2c_1 + 2c_2 = -2$

$2c_1 - 3c_2 = 8$

$5c_2 = -10 \Rightarrow c_2 = -10/5 = -2$.

$c_1 - 2 = -1$

$c_1 = -1 + 2 \Rightarrow c_1 = 1$.

$\therefore c_1 = 1, \; c_2 = -2$

Substitute in general solution.

$a_n = 1 \cdot 2^n + (-2)(-3)^n$.

$a_n = (2)^n - 2(-3)^n$

Example: Solve the recurrence relation.

$a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \geq 2$.

given that $a_0 = 5$ and $a_1 = 12$.

if a_1

Sol: Given recurrence relation is.

$a_n - 6a_{n-1} + 9a_{n-2} = 0$ ——— (1)

Characteristic equation for (1) is.

$r^2 - 6r + 9 = 0$ ——— (2)

roots for eq. (2)

$\cancel{r(r+3) + 3(r-3) = 0}$ $r(r-3) - 3(r-3) = 0$

$\cancel{(r+3)(r-3) = 0}$. $(r-3)(r-3) = 0$.

$r = 3, 3$. $r_1 = 3$, $r_2 = 3$.

Now the general solution of given recurrence relation is.

$a_n = c_1 \cdot (r_1)^n + c_2 (r_2)^n$.

$a_n = c_1 \cdot (-3)^n + c_2 (3)^n$. ——— (3)

$\therefore c_1$ and c_2 are the constants.

Initial conditions are.

$a_0 = 5$ & $a_1 = 12$.

if $a_0 = 5$ when $n = 0$.

Substitute in eq. (3)

$a_0 = c_1 \cdot (-3)^0 + c_2 (3)^0$

$c_1 + c_2 = 5$ ——— (4)

Scanned with OKEN Scanner

If $a_1 = 12$, where $n = 1$.

$$a_1 = c_1 (+3)^1 + c_2 (3)^1$$

$$+3c_1 + 3c_2 = 12 \text{ --- } (5)$$

$$\downarrow \quad c_1 + c_2 = 4 \text{ --- } (5)$$

from (4) & (5).

$$\cancel{3c_1 + 3c_2 = 15}$$

$$\cancel{3c_1 + 3c_2 = 12}$$

$$\cancel{6c_2 = 27}$$

$$\cancel{c_2 = 27/6 =}$$

$$\cancel{c_1 + c_2 = 5}$$

$$\cancel{c_1 + c_2 = 4}$$

Roots are equal and real. The general solution for this is.

$$a_n = (c_1 + c_2 n) \cdot r^n$$

$$= (c_1 + c_2 n) \cdot 3^n \text{ --- } (3)$$

$a_0 = 5$, $a_1 = 12$.

$$a_0 = (c_1 + c_2 \cdot 0) \cdot 3^0$$

$$c_1 + 0 = 5 \Rightarrow c_1 = 5$$

$$a_1 = (c_1 + c_2 \cdot 1) \cdot 3^1$$

$$3c_1 + 3c_2 = 12$$

$$c_1 + c_2 = 12/3 = 4$$

$$5 + c_2 = 4 \Rightarrow c_2 = 4 - 5 = -1$$

Scanned with OKEN Scanner

Substitute the constant values in the equation

$$a_n = (5 + (-1)\cdot n)\cdot 3^n.$$

$$\boxed{a_n = (5 - n)\cdot 3^n.}$$

$a_0 = 5$

$n = 0.$

Check the equation is correct or not.

$a_0 = 5.$

$n = 0$

$$a_0 = (5 - 0)\cdot 3^0 = 5\cdot 1 = 5.$$

$a_1 = 12.$

$n = 1.$

$$a_1 = (5 - 1)\cdot 3^1 = 4\cdot 3 = 12.$$

Find solution is correct.

$$\boxed{a_n = (5 - n)\cdot 3^n}$$

Solu
Or

Solutions of Non-homogeneous Recurrence Relation or Inhomogeneous Recurrence Relation:

If Given Recurrence Relation is a second order non-homogeneous recurrence relation.

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} + \cdots + c_k a_{n-k} + f(n) \quad \text{—} (1)$$

∴ The general solution of relation is:

$$\boxed{a_n = a_n^{(h)} + a_n^{(p)}} \quad \text{———} (2)$$

To obtain $a_n^{(h)}$, put $f(n) = 0$ in equation (1)

$$a_n + c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} = 0 \longrightarrow (3)$$

Step 1:-

We obtain the homogeneous solution:

First we write the associated homogeneous recurrence relation, namely

$$f(n) = 0.$$

i.e $a_n + c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} \cdots + c_k a_{n-k} = 0$

Then we find the general solution, which is called the homogeneous solution.

Step 2:-

We obtain the particular solution. There is no general procedure for finding the particular solution of a Recurrence Relation.

However, if $f(n)$ has any one of the following forms,

(i) polynomial in 'n'

(ii) a constant

(iii) powers of constant,

Particular solution for given f(n):

S.NO	f(n)	form of particular solution
1	A constant, C.	A constant, d
2	A linear function $c_0 + c_1 n$	A linear function $d_0 + d_1 k$, $d_0 + d_1 k$.
3	n^2	$d_0 + d_1 k + d_2 k^2$.
4	An mth degree polynomial $c_0 + c_1 n + c_2 n^2 + \dots + c_m n^m$	An mth degree polynomial $d_0 + d_1 k + d_2 k^2 + \dots + d_m k^m$
5	$r^n, r \in R$	$d r^n$.

→ ~~Solve the following recurrence relation.~~

~~$a_n + 4a_{n-1} + 4a_{n-2} = 8^n$~~

Step 3: The general solution of the recurrence relation is the sum of the homogeneous and particular solution.

Step 4: If no initial conditions are given, then step 3 will give the solution.

If 'n' initial conditions are given, then we get 'n' equations with 'n' unknowns.

Solution.

Qn:- Solve the following Recurrence Relation.

$a_n + 4a_{n-1} + 4a_{n-2} = 8$ for $n \geq 2$ with

$a_0 = 1,\ a_1 = 2.$

Sol:- Given Recurrence Relation is

$$a_n + 4a_{n-1} + 4a_{n-2} = 8, \text{ for } n \geq 2 \quad \text{——} (1)$$

Given Recurrence Relation is a second order non-homogeneous recurrence relation.

$\therefore$ The general solution of relation (1) is

$$\boxed{a_n = a_n^{(h)} + a_n^{(p)}} \quad \text{——} (2).$$

To obtain $a_n^{(h)}$, put $f(n) = 0$ in equation (1).

$$a_n + 4a_{n-1} + 4a_{n-2} = 0 \quad \text{——} (3).$$

The characteristic ~~roots~~ equation of equation (3) is.

$$r^2 + 4r + 4 = 0. \quad \text{——} (4)$$

Characteristic roots of (4) is,

$$r^2 + 2r + 2r + 4 = 0$$

$$r(r+2) + 2(r+2) = 0$$

$$r+2 = 0 \quad \& \quad r+2 = 0$$

$$r = -2, -2.$$

Here the roots are equal and real.

The general solution of $a_n^{(h)}$ is.

$$\boxed{a_n^{(h)} = (A + Bn) r^n}.$$

$$a_n^{(h)} = (A + Bn)(-2)^n.$$

obtain the $a_n^{(P)}$ will be calculated.

$$a_n^{(P)} = A_0 \quad \text{—— (5)}$$

equations (4) & (5) are substituted in eq (1).

$$a_n = a_n^{(h)} + a_n^{(P)}$$

$$a_n = (A + Bn)(-2)^n + A_0 \quad \text{—— (6)}$$

for obtaining A_0 value, eq (5) is substituted in eq (1)

$$a_n + 4a_{n-1} + 4a_{n-2} = 8$$

$$A_0 + 4A_0 + 4A_0 = 8$$

$$9A_0 = 8$$

$$\boxed{A_0 = 8/9}$$

$\cancel{d + 4(d-1) + 4(d-2) = 8}$

$\cancel{d + 4d - 4 + 4d - 8}$

$\cancel{9d - 12 = 8}$

$\cancel{9d = 8 + 12}$

$\cancel{d = 20/9}$

Now Substitute A_0 value in equation (6)

$$a_n = (A + Bn)(-2)^n + 8/9 \quad \text{—— (7)}$$

Initial Conditions $a_0 = 1$, $a_1 = 2$ ✓

if $n = 0$.

Substitute $n = 0$ in eq (7).

$$a_0 = (A + B \times 0)(-2)^0 + 8/9.$$

$$1 = A + \frac{8}{9}$$

$$A = 1 - \frac{8}{9} = \frac{9-8}{9} = 1/9.$$

if $n = 1$.

Substitute $n = 1$ in eq (7)

$$a_1 = (A + B \times 1)(-2)^1 + 8/9.$$

$$2 = \cancel{2A} - 2B + 8/9$$

Scanned with OKEN Scanner

$$-2A - 2B = 2 - 8/9$$

$$\cancel{-2B = 2A + 2 - \frac{8}{9}}$$

$$\cancel{= \frac{2}{9} - \frac{8}{9} + 2 = \frac{-5}{9} + 2}$$

$$\cancel{= \frac{-10+9}{18} = \frac{1}{18}}$$

$$\cancel{-2B = \frac{-1}{9}}$$

$$\cancel{-2B = \frac{-5+18}{9} = \frac{13}{9}}$$

$$-2A - 2B = \frac{18-8}{9} = \frac{10}{9}$$

$$-2B = \frac{10}{9} + 2A = \frac{10}{9} + 2\left(\frac{1}{9}\right)$$

$$= \frac{10+2}{9} = \frac{12}{9}$$

$$2B = \frac{-12}{18} = \frac{-6}{9} = \frac{-2}{3}$$

$$\boxed{B = \frac{-2}{3}}$$

A & B values substituted in eq. (2)

$$\boxed{a_n = \left(\frac{1}{9} + \left(\frac{-2}{3}\right) n\right)(-2)^n + \left(\frac{8}{9}\right)}$$

Ex:- (Problem).

$$a_n = 3a_{n-1} + 2n \text{ with } a_1 = 3.$$

$$a_n = 3a_{n-1} + 2^n$$

Ex: Solve the Recurrence relation $a_n = 3a_{n-1} + 2^n$ with initial condition $a_0 = 1$.

Hence

Sol: Given Inhomogeneous solution is.

$$a_n - 3a_{n-1} = 2^n \quad \text{——} \quad (1)$$

The general solution for the given RR is

$$\boxed{a_n = a_n^{(h)} + a_n^{(p)}}$$

(i) The associated homogeneous equation is

$$a_n - 3a_{n-1} = 0 \quad \text{——} \quad (2).$$

Characteristic equation of (2) is

$$\cancel{r^2} - \cancel{3r} = 0. \quad r - 3 = 0$$

$$\Rightarrow \cancel{r^2} = 3.$$

The homogeneous equation is.

$$\boxed{a_n^{(h)} = c_1 3^n.} \quad \text{——} \quad (3).$$

(ii) The R.H.S of the recurrence relation is 2^n and 2 is not the characteristic root. let the particular solution of the recurrence relation be.

$$\boxed{a_n = \cancel{A} \cdot A \cdot 2^n} \quad \text{——} \quad (4)$$

Using this equation in the given recurrence relation in (1).

we get.

$$A 2^n - 3 \times A 2^{n-1} = 2^n.$$

$$A 2^n - \frac{3A \times 2^n}{2} = 2^n.$$

$$\cancel{2^n}\left(A - \frac{3}{2}A\right) = \cancel{2^n}$$

$$A - \tfrac{3}{2}A = 1.$$

$$\frac{2A - 3A}{2} = 1$$

$$-A = 1 \Rightarrow \boxed{A = -2.}$$

Hence the particular equation is

$$a_n^{(P)} = (-2)(2)^n = -2^{n+1}$$

Hence the general solution is:

$$a_n = a_n^{(h)} + a_n^{(P)}$$

$$\Rightarrow \boxed{a_n = c_1 3^n - 2^{n+1}}$$

Using the conditions:

$a_0 = 1$, Here $n=0$, we get.

$$a_0 = c_1 3^0 - 2^{0+1}$$

~~$\boxed{a_0 = 3c_1 - 2^{0+1}}$ $\Rightarrow 1 = 3c_1 - 2$.~~

~~$1 = 3c_1 - 2$.~~

~~$3c_1 = 1+2 \Rightarrow c_1 = 3/3 = 1$.~~

$$a_0 = c_1 3^0 - 2^{0+1}$$

$$1 = c_1 - 2.$$

$$\boxed{c_1 = 3.}$$

The required solution is $a_n = 3(3^n) - 2^{n+1}$

i.e $\boxed{a_n = 3^{n+1} - 2^{n+1}}$

This is required solution.

Ex: Solve the Recurrence relation

$a_n = 2a_{n-1} + 2^n$, $a_0 = 2$.

Sol: The given recurrence relation is:

$$\boxed{a_n - 2a_{n-1} = 2^n} \quad \text{——} (1)$$

(i) The associated homogeneous equation is

$$a_n - 2a_{n-1} = 0 \quad \text{——} (2)$$

The characteristic equation is

$$r - 2 = 0$$

$$\boxed{r = 2}$$

The homogeneous solution is:

$$\boxed{a_n = c_1 2^n} \quad \text{——} (3)$$

(ii) Since the R.H.S of the recurrence relation is 2^n and 2 is the characteristic root. Let

$a_n = n \times A 2^n$ — be a particular solution of the Recurrence relation.

Using this equation in the given relation, we get:

$$n \times A 2^n - 2A(n-1) \cdot 2^{n-1} = 2^n,$$

$$An2^n - \cancel{2An\,2^{n-1} + 2A\,2^{n-1}} = 2^n,$$

$$\cancel{An2^n - 2A(n}$$

$$nA2^n - A(n-1)\,2^{n-1+1} = 2^n.$$

$$nA2^n - A(n-1)2^n = 2^n.$$

$$A\cancel{2^n}(n - (n-1)) = \cancel{2^n}$$

$$A(\cancel{n} - \cancel{n} + 1) = 1 \qquad \boxed{A = 1}$$

$$\boxed{a_n^{(P)} = n 2^n.}$$

Hence the general solution is.

$$a_n = a_n^{(h)} + a_n^{(P)}.$$

$$\Rightarrow \boxed{a_n = c_1 2^n + n 2^n}$$

Given that $a_0 = 2$.

$$a_0 = c_1 2^0 + 0 \cdot 2^0$$

$$2 = c_1 \Rightarrow \boxed{c_1 = 2}$$

Therefore the required solution is.

$$a_n = 2 \cdot (2^n) + n(2^n).$$

$$\boxed{a_n = 2^{n+1} + n \cdot 2^n.}$$

3

Ex3 Solve the following recurrence relation.

$$a_{n+1} - 2a_n = 2^n, \quad n \geq 0 \text{ with } a_0 = 1.$$

Sol:- Given recurrence relation

$$a_{n+1} - 2a_n = 2^n \quad \text{——— (1)}$$

The given RR is first order non-linear (or) non homogeneous recurrence relations.

The General Solution is,

$$a_n = a_n^{(h)} + a_n^{(P)} \quad \text{——— (2)}$$

$+$

$3n^2$.

$= 3n^2$

$= 3$.

Example ①

To obtain $a_n^{(h)}$, put $f(n) = 0$ in eq ①.

$$a_{n+1} - 2a_n = 0.$$

$$a_{n+1} = 2a_n \quad \text{③}.$$

The general solution of homogeneous first order recurrence relation is.

$$a_n = c^n \cdot a_0 \quad \text{④}$$

If $a_n = c^n$ then

$$a_{n+1} = c^{n+1}$$

$$2 \cdot a_n = c^{n+1}.$$

$$2 \cdot c^n = c^n \cdot c$$

$$2 = c. \qquad \boxed{\therefore c = 2}$$

c value can be substituted in equation ④.

$$a_n = 2^n \cdot a_0$$

$$\boxed{\therefore a_n^{(h)} = 2^n \cdot a_0}$$

Step 2: To obtain $a_n^{(p)}$, put $f(n) = 2^n$.

$$a_{n+1} - 2a_n = 2^n.$$

It is in the form of $f(n) = (b)^n$.

$f(n) =$

Example:

① $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2$.

② Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with initial conditions $a_0 = 1, a_1 = 6$.

③ $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, $a_0 = 2, a_1 = 5, a_2 = 15$.

④ $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$.

$a_0 = 1, a_1 = -2, a_2 = -1$.

⑤ $a_n = 3a_{n-1} + 2n$, $a_1 = 3$. (root: $r_1 = 3$).

⑥ $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$ (root: $r_1 = 3, r_2 = 2$)

when = $F(n) = 3^n$

$F(n) = n3^n$

$F(n) = n^2 2^n$ $\quad r_1 = 3$, multiplicity 2.

$F(n) = (n^2+1)3^n$ $\rightarrow (d_0 + d_1 n + d_2 n^2) =$

⑦ $a_n - 7a_{n-1} + 10a_{n-2} = 4^n$.

⑧ solve $a_n - 4a_{n-1} + 4a_{n-2} = (n+1)^2$. $a_0 = 0, a_1 = 1$

Ex: $F(n) = (n+1) \cdot 2^n \rightarrow n^2 d \cdot 2^n$.

$\downarrow (d_1 + d_2 n)$

$F(n) = 3n + 2^n = a_n^{(P_1)} + a_n^{(P_2)}$.

$\downarrow (d_0 + d_1 n)$ $\quad \downarrow d \cdot 2^n$

Ex: Solve the RR $a_n = 4a_{n-1} - 4a_{n-2} + 3n + 2^n$.
where $a_0 = 1$, $a_1 = 1$.

Sol:- Given Recurrence Relation is.

$$a_n = 4a_{n-1} - 4a_{n-2} + 3n + 2^n.$$

$$a_n - 4a_{n-1} + 4a_{n-2} = 3n + 2^n \quad \text{—} (1)$$

Step1: Associated Homogeneous Recurrence Relation is

$$a_n - 4a_{n-1} + 4a_{n-2} = 0 \qquad (\because f(n) = 0).$$

$$r^2 - 4r + 4 = 0.$$

$$(r-2)^2 = 0 \qquad r - 2 = 0$$
$$r - 2 = 0$$

$$\therefore r = 2, 2.$$

roots are equal & real so. General homogeneous recurrence relation is.

$$a_n^{(h)} = (d_0 + d_1 n) 2^n. / (c_0 + c_1 n) 2^n.$$

Step2: R.H.S $\Rightarrow 3n + 2^n$.

particular solution: $a_n^{(P_1)} + a_n^{(P_2)}$.

Since the part of the R.H.S is $3n$ i.e a linear polynomial let.

$$a_n^{(P_1)} = d_0 + d_1 n. \quad \text{—} (2).$$

Substitute (2) in eq. (1)

$$(d_0 + d_1 n) - 4(d_0 + d_1(n-1)) + 4(d_0 + d_1(n-2)) = 3n$$

$$d_0 + d_1 n - \cancel{4d_0} - \cancel{4d_1 n} + 4d_1 + \cancel{4d_0} + \cancel{4d_1 n} - 8d_1 = 3n$$

$\cancel{d_0 + 8d_1 + 5d_1 = 3n}$. $\quad d_0 + d_1 n - 4d_1 = 3n$.

$d_0 +$

$$(d_0 - 4d_1) + d_1 n = 3n.$$

Scanned with OKEN Scanner

Equating the co-efficients of n both sides we get:

$$d_1 = 3$$

$$d_0 - 4d_1 = 0$$

$$d_0 - 4*3 = 0$$

$$d_0 - 12 = 0 \Rightarrow d_0 = 12.$$

$\therefore$ Therefore, particular solution corresponding to $3n$ is:

$$\boxed{a_n^{(P_1)} = 12 + 3n.}$$

next, part of the R.H.S is 2^n and 2 is the double root of the

$$a_n^{(P_2)} = d \cdot 2^n \cdot n^2 \quad \text{—— (3)}$$

Substitute (3) in eq. (1)

$$\cancel{d \cdot n^2 2^n - 4d2^{n-1}(n-1)^2 + 4d2^{n-2}(n-2)^2 = d \cdot 2^n}$$

$$\cancel{d \cdot n^2 2^n - [4 \cdot 2^n (n^2+1-2n)] + [4 2^n (n^2+4-4n)] = 2^n}$$

$$\cancel{d \cdot n^2 2^n - 4n^2 2^n - 4 \cdot 2^n + 8n2^n + 4n^2 2^n + 16 \cdot 2^n - 16n2^n} \text{ going}$$

$$\cancel{d \cdot n^2 2^n + 12 \cdot 2^n - 8n2^n = 2^n} \quad \cancel{= 2^n}$$

$$d \cdot n^2 2^n - 4d \cdot 2^{n-1}(n-1)^2 + 4d \cdot 2^{n-2}(n-2)^2 = 2^n.$$

$$d \cdot n^2 \cdot 2^n - \frac{4d}{2} \cdot (n^2+1-2n) \cdot 2^n + \frac{4d}{4} \cdot (n^2+4-4n) \cdot 2^n = 2^n.$$

$$d \cdot n^2 \cdot 2^n - 2^n(2dn^2 + 2d - 4nd) + 2^n(dn^2 + 4d - 16nd) = 2^n.$$

$$2^n(\cancel{dn^2} - 2dn^2 - 2d + 4nd + \cancel{dn^2} + 4d - 16nd) = 2^n. \quad nd_2 t$$

$$2 = 3n^2.$$

$$2d - 12nd = 1$$

$n = 0$

$$2d = 1 \Rightarrow d = 1/2.$$

$$d_2 = 3n^2.$$

$$d_2) = 3.$$

Scanned with OKEN Scanner

$$a_n^{(P_2)} = \frac{1}{2} n^2 2^n$$

$$= n^2 2^{n-1}$$

$\therefore$ The particular solution is.

$$a_n^{(P)} = a_n^{(P_1)} + a_n^{(P_2)}$$

$$\boxed{= 12 + 3n + n^2 2^{n-1}}$$

Hence the general solution is,

$$\boxed{a_n = (d_0 + d_1 n)\cdot 2^n + 3n + n^2 2^{n-1}}$$

Solve the recurrence relation

$$a_n + 5a_{n-1} + 6a_{n-2} = 3n^2.$$

Sol: Given Recurrence relation is

$$a_n + 5a_{n-1} + 6a_{n-2} = 3n^2 \longrightarrow (1)$$

Step 1:- Associated homogeneous recurrence relation.

$$a_n + 5a_{n-1} + 6a_{n-2} = 0 \longrightarrow (2)$$

$$r^2 + 5r + 6 = 0$$

$$r(r+2) + 3(r+2) = 0.$$

$$r+2 = 0 \qquad r+3 = 0$$

$$r = -2, -3.$$

roots are real & distinct. so general homogeneous solution is,

$$\boxed{a_n^{(h)} = d_0(-2)^n + d_1(-3)^n}$$

Step 2:- find the particular solution. R.H.S of given RR is. $3 \cdot n^2$.

$$\Rightarrow (d_0 + d_1 n + d_2 n^2) \longrightarrow (3)$$

substitute eq (3) in eq. (1).

$$(d_0 + d_1 n + d_2 n^2) + 5(d_0 + d_1(n-1) + d_2(n-1)^2) + 6(d_0 + d_1(n-2) + d_2(n-2)^2) = 3n^2.$$

$$d_0 + d_1 n + d_2 n^2 + 5d_0 + 5nd_1 - 5d_1 + 5d_2 n^2 + 5d_2 - 10nd_2 + 6d_0 + 6nd_1 - 12d_1 + 6d_2 n^2 + 4d_2 - 4nd_2 = 3n^2.$$

$$12d_2 n^2 + 12d_0 - 17d_1 + nd_1 - 14nd_2 + 4d_2 = 3n^2.$$

$$12d_2 n^2 + 12d_0 - 17d_1 + 4d_2 + n(d_1 - 14d_2) = 3n^2$$

$$12 d_2 = 3$$
$$d_2 = 3/12 = 1/3.$$

$$d_1 - 14 d_2 = 0.$$
$$d_1 - \frac{14}{3} = 0 \Rightarrow d_1 = \frac{14}{3}.$$

$$12 d_0 - 17 d_1 + 4 d_2 = 0.$$
$$12 d_0 - 17 \cdot \frac{14}{3} + 4 \cdot \frac{1}{3} = 0.$$

$$\begin{array}{r} 17 \\ \times\ 14 \\ \hline 98 \\ 17\ \ \\ \hline 268 \end{array}$$

$$12 d_0 - \frac{268}{3} = \frac{4}{3}$$
$$12 d_0 = \frac{4}{3} + \frac{268}{3}$$
$$= \frac{272}{3} \Rightarrow d_0 = \frac{272}{3} - 12.$$

$a_n^{(P)}$. R.H.S. constant $= d$.
$$d + 5d + 6d = 3.$$
$$12d = 3. \Rightarrow d = 3/12 = 1/4.$$

Another one n^2.
$$\boxed{d_0 + d_1 n + d_2 n^2.}$$

$$(d_0 + d_1 n + d_2 n^2) + 5(d_0 + d_1 (n-1) + d_2 (n-1)^2) +$$
$$6(d_0 + d_1 (n-2) + d_2 (n-2)^2) = n^2.$$

$$d_0 + d_1 n + d_2 n^2 + 5(d_0 + d_1 n - d_1 + n^2 d_2 + 4d_2 - 4n d_2).$$
$$6(d_0 + d_1 n - 2d_1 + d_2 n^2 + 4d_2 - 4n d_2) = n^2$$

$d_0 + d_1 x + d_2 x^2 + 5d_0 + 5d_1 x - 5d_1 + 5x^2 d_2$

$+ 20d_2 - 20x d_2 + 6d_0 + 6d_1 x - 12d_1$

$+ 6d_2 x^2 + 24d_2 - 24x d_2 = x^2.$

$x^2 d_2 + x^2(d_2 + 5d_2 + 6d_2)$

$d_2 - 13d_2 = 1$

$d_2 = 1/13.$