Predicate Logic (UNIT-II).

Logic Two types
- propositional logic
- predicate logic

Propositional logic:

The logic that deals with the analysis of propositions or statements is called propositional logic.

Predicate logic:

The logic that deals with the analysis of predicates in any statement is called predicate logic/predicate calculus.

Here we discussed on:-

1) What is Predicate?
2) How we are assigning the truth value for predicates.
3) How we are representing the predicates.

Example: "x is an even number" — Two parts.

x — Variable x. — subject/object/variable — first part

is an even number — predicate. —

Predicate:- The property of the subject is called predicate.

Subject — lower case letters.
predicates — capital letters } we are represent.

E(x) → subject — represented with in the parenthesis
→ Predicate. subject is always written.

→ If you are assigning any number to the [illegible] it becomes a statement.

if $x = 10$.

$E(10)$ — 10 is a even number — Truth value (T)

if $x = 5$

$E(5)$ — 5 is a even number — Truth value (F).

→ In general,

"x is greater than 10"

The above stmt has two parts.

(i) first part is "variable x"

It is the subject of the statement.

(ii) The second part is "is greater than 10".

which refers to a property that can have is called the predicates.

⇒ We can ~~determine~~ denote the statement

"x is a greater than 10".

by the notation $P(x)$.

'p' is predicate — is a greater than 10.

x is a variable x.

Ex: "x is taller than y"

predicate — is taller than

subject — x, y.

$T(x, y)$. — This predicate belongs to 2 subjects. This type is called 2 place predicate.

T → predicate

(x, y) → subjects.

1 - 1

1- place predicate = $H(x)$.

predicate that belongs to one subjects.

2-place predicate = $T(x,y)$

Predicate that belongs to 2 subjects.

3-place predicate = $T(x,y,z)$.

x is taller than y and z.

predicate belongs to 3 subjects.

Note:- predicate belongs to n subjects is called n-place predicates.

$T(1,2,3 ---- n)$ — n place predicates.

~~compound statement in predicate logic~~;

M-place predicates

A predicate requiring 'm' names is called an m-place predicate.

Example:-

1. Amulya is a student

$S(a)$ — one place predicate.

2. Naveen is taller than Amul.

$T(n,a)$ — Two place predicate

3. Raju sits between the Rama & Ravi — 3 place Predicate.

Note:- If 's' is a n-place predicate and $a_1, a_2 --- a_n$ are the names of the objects then.

$s(a_1, a_2, --- a_n)$ is a statement.

4. "Green and Miller played bridge against John & Smith" — four place Predicates.

$B(G, M, J, S)$.

Compound statement in predicate logic

Consider the statement:

(i) John is a Bachelor.

(ii) This painting is red.

→ Variable – John.

Predicate – is a Bachelor.

$\therefore B(J)$.

→ Variable – This painting.

Predicate – is red.

$\therefore R(P)$.

The compound stmts can be obtained from the above two statements.

(i) John is a Bachelor and this painting is red.

$$B(J) \wedge R(P).$$

(ii). John is a Bachelor or this painting is red.

$$B(J) \vee R(P)$$

(iii) If John is a Bachelor then this painting is red.

$$B(J) \rightarrow R(P).$$

Note: Connectives can be used to form the compound stmts from the given statements.

→ The truth value of the compound predicate statement is depends on the simple atomic ~~predicate~~/ statement.

Quantifiers

Ex: Provide some example compound statements for the following statements:

" x is a Mortal ", " x is a Man "

Sol: $M(x)$ — x is a Man.

$H(x)$ — x is a Mortal.

1) $M(x) \wedge H(x)$
2) $M(x) \vee H(x)$.
3) $M(x) \rightarrow H(x)$.
4) $M(x) \leftrightarrow H(x)$.

Ex: let $Q(x,y)$ denote the stmt "$x = y+3$"
what are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$.

Sol: $Q(x,y)$; $x = y+3$.

$Q(1,2) \Rightarrow$ if $x = 1$, $y = 2$.

$1 = 2+3 \Rightarrow 1 = 5$ which is false.

$\therefore Q(1,2)$ is false.

$Q(3,0) \Rightarrow$ if $x = 3$, $y = 0$.

$\cancel{0 = 3+3 \Rightarrow 0 = 6}$

$3 = 0+3 \Rightarrow 3 = 3$ which is true.

The truth values of $Q(1,2)$ & $Q(3,0)$ are F and T, respectively.

Quantifiers

Consider the following statements or propositions.

$\forall$ (i) All birds have wings

$\exists$ ii) Some men are tall.

$\forall$ iii) No air ballon is perfectly round.

$\forall$ iv) For every integer "n" n^2 is a non - negative integer.

$\exists$ v). There exists a real number whose square is equal to itself.

$\Rightarrow$ The above statements involve certain words.

"All"

"Some"

"non one"

"There exists"

"every".

that are associated with the idea of a quantity. Such words are called "Quantifiers".

There are two types of Quantifiers

(i) Universal Quantifier.

(ii) Existential Quantifier.

Universal Quantifier:

The Quantifier "All" is called universal Quantifier and ~~it is~~ it is denoted by "$\forall$".

The following phrases have same meaning.

- for all
- for every
- for each
- everything
- each thing.

represents universal Quantifier.

Existential Quantifier:

The quantifier "some" is called existential quantifier and it is represented "$\exists$".

The symbol $\exists$ represents the following phrases same meaning.

- There exists
- There is atleast
- There is an.

represents Existential Quantifier.

Examples on Quantifiers:

(1) write the following statements in symbolic form.

(a) some thing is good.

(b) everything is good.

(c). Nothing is good

(d). Something is not good.

Sol: let us consider.

$G(x)$: x is good.

(a) Some thing is good.

There is atleast one 'x' such that x is good. $\exists x\, G(x)$.

Negation

(b) for all x, such that x is good.

$\forall x\, G(x)$.

(c) for all x, such that x is not good.

$\forall x \sim G(x)$.

(d) There is alleast one x such that x is not good.

$\exists x \sim G(x)$.

Ex:- write the following statements in symbolic form.

(a) All men are good.

(b) No men are good.

(c) Some men are good.

(d) Some men are not good.

Sol:- let us consider.

$M(x)$: x is a man

$G(x)$: x is good.

(a) for all x, if x is man, then x is good.

$\forall x (M(x) \rightarrow G(x))$.

(b) for all x, if x is a man, then x is not good.

$\forall x (M(x) \rightarrow \sim G(x))$.

(c) There is atleast one x, x is a man and x is good.

$\exists x (M(x) \wedge G(x))$.

(d) There is atleast one x, x is a man and x is not good

$\exists x (M(x) \wedge \sim G(x))$.

Negation of a Quantified Statement:

To construct the negation of a quantified statement change the quantifier from universal to existential & existential to universal quantifier and also replace the given stmt by its negation.

Statement	Negation of statement
① $\forall(x)\ P(x)$.	$\sim(\forall(x).\ P(x))$.
	$\Leftrightarrow \exists(x) \sim P(x)$.
② $\exists(x)\ P(x)$.	$\sim(\exists(x).\ P(x))$.
	$\Leftrightarrow \forall(x).\ \sim P(x)$.
③ $\forall(x) \sim P(x)$.	$\sim(\forall(x). \sim P(x))$.
	$\Leftrightarrow \exists(x).\ P(x)$.
④ $\exists(x). \sim P(x)$	$\sim(\exists(x). \sim P(x))$.
	$\Leftrightarrow \forall(x).\ P(x)$.

Example: Find out the negation of the following quantified statement.

(i) All equilateral triangles are isosceles.

(ii). Some equilateral triangles are not isosceles.

Sol: $T(x)$: x is a equilateral tringle.

$I(x)$: x is a Isosceles.

(i).

(i) for all x, if x is a an equilateral triangle then x is isosceles.

$$\forall(x)\,(T(x) \to I(x)).$$

Negation of the above statement:

$$\sim\left(\forall(x)\,[T(x) \to I(x)]\right).$$

$$\exists(x).\ \sim[T(x) \to I(x)].$$

$$\exists(x).\ \sim(\sim T(x) \vee I(x)).$$

$$\exists(x).\ T(x) \wedge \sim I(x).$$

(ii). $\exists(x)[T(x) \wedge \sim I(x)]$

negation of the above statement is

$$\sim\left[\exists(x).\,[T(x) \wedge \sim I(x)]\right].$$

$$\forall(x).\ \sim[T(x) \wedge \sim I(x)]$$

$$\forall x.\ (\sim T(x) \vee I(x)).$$

Represent

3) write

(

Representing the statements in Symbolic form

3) Write down the Quantified Statements in Symbolic form.

(a) All monkeys have tails

(b) No ~~moneys~~ Monkeys has a tail

(c) Some monkeys have tails.

(d) Some monkeys have no tails.

Sol: let us consider:

$M(x)$: x is a monkey.

$T(x)$: x has a tail.

(a) for all x, If 'x' is a monkey then x has a ~~tail~~ tail.

$\forall x\,(M(x) \rightarrow T(x))$.

(b) for all x, If x is a monkey then x has not a tail.

$\forall x\,(M(x) \rightarrow \sim T(x))$.

(c) There is atleast one x, such that x is a monkey and x has a tail.

$\exists x\,(M(x) \wedge T(x))$

(d) There is atleast one x, such that x is a monkey and x has a ~~not a tail~~ not a tail.

$\exists x\,(M(x) \wedge \sim T(x))$.

4) Write down the Quantified Statements in symbolic form.

(a) Some people who trust others are rewarded.

(b) Some one is teasing

(c) no one is ambitious.

(d) If any one is good, then John is good.

(e) It is not true that, all ~~leads~~ roads leads to Delhi

Sol: let us consider

$P(x)$: x is people

$T(x)$: x trust others

$R(x)$: x is rewarded

$T_1(x)$: x is teasing

$A(x)$: x is ambitious

$G(x)$: x is good

$R_1(x)$: x is a road

$L(x)$: x leads to delhi

(a). $\exists(x).(P(x) \land T(x) \land R(x))$.

(b) $\exists(x)\,(P(x) \land T_1(x))$.

(c) $\forall x\,(P(x) \to \sim A(x))$.

(d). $\forall(x)(\,P(x) \land G(x) \to G(\text{John}))$.

(e). $\sim\forall(x)\,(R_1(x) \to L(x))$.

Free variable, bound variable, scope of the Quantifier

Free variable:

An occurence of a variable that is not bounded by Quantifier (either Universal Quantifier or existential Quantifier). is said to be free variable.

Bound Variable:

An occurrence of a variable that is bounded by a Quantifier (either a universal Quantifier or existential Quantifier) is said to be bound variable.

Scope of the Quantifier:

The part of the logical expression or predicate formula to which a quantifier is applied called the scope of the Quantifier.

(or).

The scope of the Quantifier is the formula immediately following the quantifier.

Example:

"All men are tall"

$M(x)$: x is a man

$T(x)$: x is a Tall.

Sol: $\forall(x)\,(M(x) \rightarrow T(x))$.

Free variable: NIL — 0.

Bound variable: universal quantifier $\forall(x)$. — x.

Scope of the quantifier: $M(x) \rightarrow T(x)$.

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Example;

Predicate formula	Bound variable (B)	free variable (F)	Scope of the Quantifier (S).
(1) $\forall x \; P(x,y)$	x	y	$P(x,y)$.
(2) $\forall x (P(x) \to Q(x))$.	x	—	$P(x) \to Q(x)$.
(3) $\forall x (P(x) \to (\exists y) R(x,y))$	x, y	—	$\forall x$: $P(x) \to \exists y \, R(x,y)$ $\exists x$: $R(x,y)$
(4) $\forall x (P(x) \land Q(x)) \lor \forall y \, R(y)$.	x, y	—	$\forall(x)$: $P(x) \land Q(x)$ $\exists(x)$: $R(y)$.
(5) $\exists x (P(x) \land Q(x))$	x	—	$P(x) \land Q(x)$.
(6) $\exists(x) \, P(x) \land Q(x)$	First x	Second x	$P(x)$.

$\exists(x) (P(x) \land Q(x))$

$(P(x) \to Q(y))$

x, y

Universe of Discourse : The universe of discourse is also called universe, is the ~~a~~ set of objects of interest.
The propositions in the predicate logic are statements on objects of a universe. The universe is thus the domain of the (individual) variable.

Ex:- let $Q(x)$: x is greater than 10.
consider the following universes ~~of~~ for this stmt.

a: $\{11, 15, 20, 25, 30\}$
b. $\{5, 15, 20, 25, 30\}$
c. $\{5, 7, 8, 9\}$.

with universe (a). we can produce the statement.
$Q(11)$: 11 is greater than 10.
$Q(15)$: 15 " "
Similarly we can produce $Q(20)$, $Q(25)$ & $Q(30)$.
All these stmts are true.
$\therefore \forall x (Q(x))$ is a true statement in the universe.

$\Rightarrow$ with universe (b).
$Q(5)$ is false statement. while $Q(15)$, $Q(20)$, $Q(25)$ and $Q(30)$ are true statements.
so with universe (b) $\exists x (Q(x))$ is a true statement.
while $\forall (x) (Q(x))$. is a false statement.
$\therefore$ ~~$\forall (x)$~~ $(Q(x))$ is false means $\exists (x) (\sim Q(x))$ is a true statement.

With universe (c), All the statements $Q(1)$, $Q(7)$, $Q(8)$, & $Q(9)$ are false and hence

$\forall(x)(\sim Q(x))$ is true.

It follows that $\sim \exists(x)(Q(x))$ is true.

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Inference Theory for predicate calculus:

To understand the inference theory for predicate calculus, it is important to be familiar with the following rules.

1) Specification
2) Generalisation

Specification: The elimination of quantifiers can be done by rules of specification called US & ES. Universal specification & existential specification.

Generalisation: To prefix the correct quantifier can be done by rules of generalisation called UG & EG. Universal generalisation and existential generalisation.

Universal specification (US). (Rule US):

If a stmt of form $\forall x\, P(x)$ is assumed to be true, then the universal quantifier can be dropped to obtain $P(t)$ is true for an arbitrary object 't' in the universe.

In symbols, this rule is

$$\frac{\forall(x)\, P(x)}{\therefore P(t) \text{ for all } t.}$$

Rule ES: (existential specification)

If $\exists(x)(P(x))$ is assumed to be true, then there is an element t in the universe such that $P(t)$ is true.

In symbols, this rule is

$$\frac{\exists x(P(x))}{\therefore P(t) \text{ for some } t}$$

Rule UG: (Universal Generalisation)

If a statement $P(t)$ is true for each element t of the universe, then the universal quantifier may be prefixed to obtain $\forall x(P(x))$

In symbol, this rule is

$$\frac{P(t) \text{ for all } t}{\therefore \forall x\, P(x)}$$

Rule EG: (Existential Generalisation)

$P(t)$ is true for some elements in the universe. then $\exists x(P(x))$ is true

In symbols, we have

$$\frac{P(t) \text{ for some } t}{\therefore \exists x(P(x))}$$

Example:

Equivalence formulas for predicate logic:

① : $(\exists x)[A(x) \vee B(x)] \Leftrightarrow \exists(x)\, A(x) \vee \exists(x)\, B(x)$.

② $(x)\,[A(x) \wedge B(x)] \Leftrightarrow \forall(x)\, A(x) \wedge \forall(x)\, B(x)$.

③ $\sim(\exists(x)\, A(x)) \Leftrightarrow (x) \sim A(x)$.

④ $\sim(\forall(x)\, A(x)) \Leftrightarrow \exists(x) \sim A(x)$.

Verify the validity of the following argument.

Every living thing is a plant or animal, John's gold fish is alive and it is not a plant. Animals have hearts, Therefore John's gold fish has a heart.

$P(x)$: x is a plant

$A(x)$: x is a animal.

$H(x)$: x is a heart.

g: Johns gold fish.

$$\forall(x)(P(x) \vee A(x))$$
$$\sim P(g).$$
$$\forall x\,(A(x) \to H(x))$$
$$\therefore H(g).$$

	(1)	$\forall(x)(P(x) \vee A(x))$	Rule P.
	(2)	$\sim P(g)$	Rule P
{1}	(3)	$P(g) \vee A(g)$	Rule US
{2,3}	(4)	$A(g)$	Rule T,
	(5).	$\forall(x)\,(A(x) \to H(x))$.	Rule P
{5}	(6)	$A(g) \to H(g)$	Rule T, US.
{4,6}	(7).	$H(g)$.	Rule T.

$\therefore$ Thus the conclusion is valid.