Inference Calculus (UNIT-II).

Theory of Inference for statement calculus

The main aim of logic is to provide rules of inference to infer a conclusion from set of certain premises.

→ The Theory associated with such rules is known as "inference theory" because it is concerned with the inference of a conclusion from certain premises.

→ when a conclusion is derived from certain premises by using the accepted rules of reasoning, the process of derivation is called a deduction or formula proof.

→ In a formula proof, every rule of inference that is used at any stage in the derivation is acknowledged.

→ The rule of inference are criteria for determing the validity of an argument.

→ These rules are stated in terms of the forms of the statements (premises and conclusions) involved rather than in terms of the actual statements or their truth values.

therefore, the rules will be given in terms of statements formulas rather than in terms of any specific statements.

→ Any conclusion that is arrived at by following these rules is called a valid conclusion and the argument is called a valid argument.

Note:-

argument:- A set of statements which leads to a valid conclusion is called an argument.

$P \Rightarrow Q$.

"p" logically derives "Q".

P = Hypothesis or premise.

Q = conclusion or consequent

i.e $\{H_1, H_2, H_3 \ldots H_n\} \Rightarrow C$.

→ validity is checked by using two methods.

① By using truth table.

② By using formulas (Rules of inference).

① By validity using truth tables:

The method to determine whether the conclusion logically follows from the given premises by constructing the relevant truth table is called "truth table technique".

Definition:- let A and B be two statement formulas we say that "B logically follows from A" (or). "B is valid conclusion of the premise A". iff $A \to B$ is a tautology. i.e $A \Rightarrow B$. ($A \to B = T$).

By extending the above definition, we say that from a set of premises $\{H_1, H_2, H_3 \dots H_m\}$ and conclusion 'c' follows logically s.t.

$$H_1 \wedge H_2 \dots \wedge H_m \Rightarrow C.$$

Example: prove that $H_1: P \to q$, $H_2: P$, $C: q$. determine whether the conclusion 'c' valid or invalid.

Sol: we prove that $H_1 \wedge H_2 \Rightarrow C$.

i.e $(H_1 \wedge H_2) \to C$ = Tautology.

P	q	$P \to q$	$(P \to q) \wedge P$	$(P \to q \wedge P) \to q$ (o/p)
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Tautology.

$\therefore$ it is a valid conclusion.

Example:- Determine whether the conclusion 'c' is valid in the following premises. (By truth table techniques).

$H_1: P \to q$, $H_2: \sim P$, $C: q$.

Sol: we prove that $H_1 \wedge H_2 \Rightarrow C$.

i.e $(P \to q) \wedge (\sim P) \Rightarrow (q)$.

$(P \to q) \wedge (\sim P) \to (q)$ is a tautology.

Example
Determine whether the conclusion 'C' follows logically

P	q	$P\to q$	$\sim P$	$(P\to q)\wedge\sim P$	$(P\to q)\wedge(\sim P)\to q$ (q/c)
T	T	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	F

not a tautology.

$\therefore$ it is not a valid argument.

Example: $H_1: \sim q$
$H_2: P\to q$
$C: \sim P$

Determine whether it is valid or invalid.

Sol:

$\sim P$	P	q	$\sim q$	$P\to q$	$(\sim q\wedge(P\to q))$	$\sim q\wedge(P\to q)\to(\sim P)$
F	T	T	F	T	F	T
F	T	F	T	F	F	T
T	F	T	F	T	F	T
T	F	F	T	T	T	T

$\therefore$ it is valid argument.

Example: $H_1: r$, $H_2: P\vee\sim P$, $C: r$.

P	r	$\sim P$	$P\vee\sim P$	$r\wedge(P\vee\sim P)$	$(r\wedge(P\vee\sim P))\to r$.
T	T	F	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	T	F	T

$\therefore$ it is valid argument.

Example

Determine whether the conclusion 'C' follows logically from the premises H_1 and H_2 in the following cases.

① $H_1 : p \to q$, $H_2 : \sim(p \wedge q)$, $C : \sim p$.

p	q	$p \to q$	$p \wedge q$	$\sim(p \wedge q)$	① $(p \to q) \wedge \sim(p \wedge q)$	② $\sim p$	① $\to$ ②
T	T	T	T	F	F	F	T
T	F	F	F	T	F	F	T
F	T	T	F	T	T	T	T
F	F	T	F	T	T	T	T

∴ it is valid inclusion.

② $H_1 : \sim p$, $H_2 : p \leftrightarrow q$, $C : \sim(p \wedge q)$.

Sol:-

p	q	$\sim p$	$p \leftrightarrow q$	① $\sim p \wedge (p \leftrightarrow q)$	$(p \wedge q)$	② $\sim(p \wedge q)$	① $\to$ ②
T	T	F	T	F	T	F	T
T	F	F	F	F	F	T	T
F	T	T	F	F	F	T	T
F	F	T	T	T	F	T	T

③ $H_1 : p \to q$, $H_2 : q$, $C : p$.

p	q	$p \to q$	$(p \to q) \wedge q$	$((p \to q) \wedge q) \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

tautology.

∴ it is not valid argument.

Example:- Show that the conclusion 'c' follows from the premises H_1, H_2 --- in the following cases.

(a) $H_1 : P \to Q$, $C : P \to (P \wedge Q)$.

Sol:-

P	Q	$P \to Q$	$P \wedge Q$	$P \to (P \wedge Q)$	$(P \to Q) \to P \to (P \wedge Q)$
T	T	T	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	F	T	F	T	T

Tautology
valid.

$\therefore$ The 'c' follows from set of premises.

(b) $H_1 : \sim P$, $H_2 : P \vee Q$, $C : Q$.

Sol:-

P	Q	$\sim P$	$P \vee Q$	$(\sim P) \wedge (P \vee Q)$	$(\sim P \wedge (P \vee Q)) \to Q$.
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

valid.

$\therefore$ C follows from the premises H_1 and H_2 because in third row H_1 & H_2 values are T and 'c' value is T.

$\therefore$ The conclusion is valid conclusion.

(c) $H_1: \sim P \vee Q$, $H_2: \sim(Q \wedge \sim R)$, $H_3: \sim r$, $C: \sim P$.

P	Q	R	$\sim P$	$(\sim P \vee Q)$	$\sim R$	$Q \wedge \sim R$	$\sim(Q \wedge \sim r)$	$H_1 \wedge H_2 \wedge H_3$	$(H_1 \wedge H_2 \wedge H_3) \to C$
T	T	T	F	T	F	F	T	F	T
T	T	F	F	T	T	T	F	F	T
T	F	T	F	F	F	F	T	F	T
T	F	F	F	F	T	F	T	F	T
F	T	T	T	T	F	F	T	F	T
F	T	F	T	T	T	T	F	F	T
F	F	T	T	T	F	F	T	F	T
F	F	F	T	T	T	F	T	T	T

$\therefore$ it is a valid conclusion.

Example:- Determine whether the conclusion 'C' is valid in the following when H_1, H_2 --- are the premises.

(a) $H_1: P \to q$, $H_2: \sim q$, $C: \sim P$. (valid)

(b) $H_1: P \vee q$, $H_2: P \to r$, $H_3: q \to r$, $C: r$.

② Rules of Inference (without using truth table)

The truth table technique becomes tedious when the number of atomic variables present in all the formulas representing the premises and the conclusion is large.

→ To overcome this disadvantage, we need to investigate other possible methods, without using truth table.

→ we now describe the process of derivation by which one demonstrates that a particular formula is a valid consequence of a given set of premises.

→ Before, we do this, we give two rules of inference which are called Rule P and T.

Rule P:- A premise may be introduced at any point in the derivation.

Rule T:- A formula S may be introduced in a derivation if 'S' is a tautologically implied by any one or more of the preceding formulas in the derivation.

→ Before we proceed with the actual process of derivation, we list some important implications and equivalences that will be referred to ~~freq~~ frequently.

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Implications

① I_1 — $P \wedge Q \Rightarrow P$ } simplification.

② I_2 — $P \wedge Q \Rightarrow Q$ }

③ I_3 — $P \Rightarrow P \vee Q$ } addition.

④ I_4 — $Q \Rightarrow P \vee Q$ }

⑤ I_5 — $\sim P \Rightarrow P \to Q$

⑥ I_6 — $Q \Rightarrow P \to Q$

⑦ I_7 — $\sim(P \to Q) \Rightarrow P$

⑧ I_8 — $\sim(P \to Q) \Rightarrow \sim Q$.

⑨ I_9 — $P, Q \Rightarrow P \wedge Q$. — (Rule of conjunction)

⑩ I_{10} — $\sim P, P \vee Q \Rightarrow Q$. — (disjunctive syllogism)

⑪ I_{11} — $P, P \to Q \Rightarrow Q$. — (modus ponens)

⑫ I_{12} — $\sim Q, P \to Q \Rightarrow \sim P$ — (Modus tollens)

⑬ I_{13} — $P \to Q, Q \to R \Rightarrow P \to R$ — (hypothetical syllogism)

⑭ I_{14} — $P \vee Q, P \to R, Q \to R \Rightarrow R$. — (dilemma)

Equivalences

① E_1 — $\sim\sim P \Leftrightarrow P$. — double negation.

② E_2 — $P \wedge Q \Leftrightarrow Q \wedge P$. } Commutative Law.

③ E_3 — $P \vee Q \Leftrightarrow Q \vee P$. }

④ E_4 — $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$ } associative law

⑤ E_5 — $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$. }

⑥ E_6 — $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$. } distributive law.

⑦ E_7 — $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$. }

⑧ E_8 — $\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$. } De morgan's Law.

⑨ E_9 — $\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$. }

Examples

(10) E_{10} — $P \vee P \Leftrightarrow P$

(11) E_{11} — $P \wedge P \Leftrightarrow P$ } Idempotent law.

(12) E_{12} — $R \vee (P \wedge \sim P) \Leftrightarrow R$.

(13) E_{13} — $R \wedge (P \wedge \sim P) \Leftrightarrow R$.

(14) E_{14} — $R \vee (P \vee \sim P) \Leftrightarrow T$.

(15) E_{15} — $R \wedge (P \wedge \sim P) \Leftrightarrow F$.

(16) E_{16} — $P \to Q \Leftrightarrow \sim P \vee Q$. — law of Implication.

(17) E_{17} — $\sim (P \to Q) \Leftrightarrow P \wedge \sim Q$.

(18) E_{18} — $P \to Q \Leftrightarrow \cancel{\sim P \vee Q}\ \sim Q \to \sim P$; — law of Contrapositive

(19) E_{19} — $P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$.

(20) E_{20} — $\sim (P \rightleftarrows Q) \Leftrightarrow P \rightleftarrows \sim Q$.

(21) E_{21} — $P \rightleftarrows Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$.

(22) E_{22} — $P \rightleftarrows Q \Leftrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$.

Examples

① Demonstrate that 'R' is a valid inference from the premises $P \to Q$, $Q \to R$ and P.

Sol:-

$H_1: P \to Q$

$H_2: Q \to R$.

$H_3: P$

$C: R$.

1. $P \to Q$ — By rule P.

2. $Q \to R$ — By rule P.

(1,2) 3. $P \to R$ — By Rule T (transitive Property)

4. P — By rule P.

(3,4) 5. R — By rule T (rule of detachment). or Modus Ponens.

$\therefore$ R is a valid conclusion.

② Show that $r \vee s$ follows logically from the premises $c \vee d$, $(c \vee d) \to \sim h$, $\sim h \to (a \wedge \sim b)$, and $(a \wedge \sim b) \to (r \vee s)$.

Sol:-

$H_1: c \vee d$

$H_2: c \vee d \to \sim h$.

$H_3: \sim h \to (a \wedge \sim b)$

$H_4: (a \wedge \sim b) \to r \vee s$.

$C: r \vee s$.

[illegible] implied by a

① $c \vee d \rightarrow \sim h$ — rule P.

② $c \vee d$ — rule P.

(1,2) ③ $\sim h$ — rule T (Modus Ponens).

④ $\sim h \rightarrow (a \wedge \sim b)$ — rule P

(3,4) ⑤ $a \wedge \sim b$ — rule T (rule of detachment)

⑥ $a \wedge \sim b \rightarrow r \vee s$ — rule P.

(5,6) ⑦ $r \vee s$ — rule T (Modus ponens)

$\therefore$ C: $r \vee s$ is a valid conclusion.

(OR).

① $c \vee d \rightarrow \sim h$ — rule P.

② $\sim h \rightarrow a \wedge \sim b$ — rule P.

(1,2) ③ $c \vee d \rightarrow a \wedge \sim b$ — rule T (transitive property)

④ $a \wedge \sim b \rightarrow r \vee s$ — rule P

(3,4) ⑤ $c \vee d \rightarrow r \vee s$ — rule T (transitive property)

⑥ $c \vee d$ — rule P.

(5,6) ⑦ $r \vee s$ — By rule T (Modus Ponens)

$\therefore$ C: $r \vee s$ is a valid conclusion.

③ Show that $S \vee R$ is tautologically implied by a $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$.

Sol:- $H_1 : P \vee Q$

$H_2 : P \to R$

$H_3 : Q \to S$.

$\sim p \to q = p \vee q$

$\therefore C : S \vee R$.

① $P \vee Q$ —— By rule P.

② $\sim P \to Q$ —— By rule T, law of implication.

③ $Q \to S$ —— By rule P.

(2,3) ④ $\sim P \to S$ —— By rule T (transitive property)

(4) ⑤ $\sim S \to P$ —— By rule T (Law of contrapositive)

⑥ $P \to R$ —— rule P.

(5,6) ⑦ $\sim S \to R$ —— rule T (T.P).

(7) ⑧ $S \vee R$ —— rule T (law of implication)

$\therefore C : S \vee R$ is a valid conclusion.

④ Show that $M \vee N$ is a valid argument from the premises $\sim J \to (M \vee N)$, $(H \vee G) \to \sim J$, $H \vee G$.

Sol:- $H_1 : \sim J \to M \vee N$

$H_2 : H \vee G \to \sim J$

$H_3 : H \vee G$

$C : M \vee N$.

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① $H \vee G \rightarrow \sim J$ — By rule P

② $H \vee G$ — rule P.

(1, 2) ③ $\sim J$ — rule T (Modus Ponens)

④ $\sim J \rightarrow M \vee N$ — rule P.

(3, 4) ⑤ $M \vee N$ — rule T (Modus Ponens)

⑤ Show that $\sim S$ is a valid argument from the premises $P \rightarrow Q$, $(\sim Q \vee R) \wedge (\sim R)$, $\sim(\sim P \wedge S)$.

Sol:- $H_1 : P \rightarrow Q$

$H_2 : (\sim Q \vee R) \wedge (\sim R)$

$H_3 : \sim(\sim P \wedge S)$.

$C : \sim S$.

$P \vee \sim S$.
$\sim P \rightarrow \sim S$.
$\sim R \rightarrow \sim P$
$\sim R \rightarrow \sim S$
$\sim R$
$\boxed{\sim S}$.

$P \wedge Q = P$
$F \wedge Q = F$

① $P \rightarrow Q$ — rule P

② $(\sim Q \vee R) \wedge (\sim R)$ — rule P.

(2) ③ $\sim Q \vee R$ — rule T, simplification

(3) ④ $Q \rightarrow R$ — rule T, implication.

(1, 4) ⑤ $P \rightarrow R$ — rule T, (T.P).

II nd method

$\sim P \rightarrow \sim S$

⑥ $\underline{\sim(\sim P \wedge S)}$ — rule P — $P \vee \sim S$

$P \vee \sim S$
$\sim(\sim P \wedge \sim\sim S)$
$\sim(\sim P \wedge S)$

(6) ⑦ $\sim(\sim P \wedge \sim\underline{\sim S})$ — rule T, law of double negation.

$\sim P \vee \sim S$

(7) ⑧ $\sim(\underline{\sim(\sim P \rightarrow \sim S)})$ — rule T (implication) $\sim(P \rightarrow Q)$.

$P \wedge \sim Q$.

(8) ⑨ $\sim(\sim(\sim S))$. — rule T, $\sim(P \rightarrow Q) \Rightarrow \sim Q$

(9) ⑩ $\sim S$ — rule T, double negation.

$\sim R \rightarrow \sim P$
$\sim R$
$\Rightarrow \sim P$
$\sim P \wedge \sim S$

∴ Valid conclusion.

(6) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \to R$, $P \to M$ and $\sim M$.

Sol:- $H_1 : P \vee Q$

$H_2 : Q \to R$

$H_3 : P \to M$

$H_4 : \sim M$.

$\therefore C : R \wedge (P \vee Q)$.

$\sim P$
$\sim P \to Q$
Q

$\sim P \; 2 \; \sim P \to Q$
$\sim P$
$\sim Q$

$\sim Q$
$\sim P$

(1) $P \to M$ — rule P

(2) $\sim M$ — rule P.

(1,2) (3) $\sim P$ — Rule T, (Modus Tollens)

(4) $P \vee Q$ — rule P

(3,4) (5) Q. — rule T (disjunctive syllogism)

(6) $Q \to R$. — rule P.

(5,6) (7) R — rule T, (Modus Ponens)

(4,7) (8) $R \wedge (P \vee Q)$ — rule T (Rule of conjunction)

(7) Show that $\sim Q, P \to Q \Rightarrow \sim P$.

Sol:- $H_1 : \sim Q$, $H_2 : P \to Q$, $C : \sim P$.

(1) $P \to Q$ — rule P.

(1) (2) $\sim Q \to \sim P$ — rule T, (rule of contrapositive)

(3) $\sim Q$ — rule P.

(2,3) (4) $\sim P$ — rule T, Modus Ponens.

Show that the validity of the following arguments for the given premises and the conclusion.

① $\sim(p \wedge \sim q), \sim q \vee r, \sim r \Rightarrow \sim p$.

sol:- $H_1 : \sim(p \wedge \sim q)$

$H_2 : \sim q \vee r$

$H_3 : \sim r$

$C : \sim p$.

① $\sim q \vee r$ — rule P.

(1) ② $q \to r$ — rule T, (law of implication)

③ $\sim r$ — rule P.

(2,3) ④ $\sim q$ — rule T, Modus Tollens.

⑤ $\sim(p \wedge \sim q)$ — rule P.

(5) ⑥ $\sim p \vee q$ — rule T, Demorgan's.

(6) ⑦ $p \to q$ — rule T, law of implication

$\to \sim q \to \sim p$

(7,4) ⑧ $\sim p$ — rule T, (modus Tollens)

∴ It is valid.

② $(a \to b) \wedge (a \to c), \sim(b \wedge c), d \vee a \Rightarrow d$.

sol: $H_1 : (a \to b) \wedge (a \to c)$

$H_2 : \sim(b \wedge c)$

$H_3 : d \vee a$.

$\therefore C : d$.

① $(a \to b) \wedge (a \to c)$ —— rule P

(1) ② $a \to b$ —— rule T, simplification

(1) ③ $a \to c$ —— rule T, "

(2) ④ $\sim b \to \sim a$ —— rule T, contrapositive.

(3) ⑤ $\sim c \to \sim a$ —— rule T, "

(4,5) ⑥ $(\sim b \vee \sim c) \to \sim a$ —— rule of proof by cases.

(6) ⑦ $\sim (b \wedge c) \to \sim a$ —— rule T, demorgan's law

⑧ $\sim (b \wedge c)$ —— rule P

(7,8) ⑨ $\sim a$ —— rule T, R.D or M.P.

⑩ $d \vee a$ —— rule P.

(10) ⑪ $\sim d \to a$ —— rule T, implication

⑫ $\sim a \to d$ —— contrapositive.

(9,12) ⑬ d —— rule T, R.D

③ Show that $(a \vee b)$ follows logically from the premises $p \vee q$, $(p \vee q) \to \sim r$, $\sim r \to s \wedge \sim t$ and $s \wedge \sim t \to a \vee b$.

Sol:- H_1 : $p \vee q$

H_2 : $p \vee q \to \sim r$

H_3 : $\sim r \to s \wedge \sim t$

H_4 : $s \wedge \sim t \to a \vee b$

$\therefore a \vee b$

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① $p \vee q$ — rule P.

② $(p \wedge q) \to \sim r$ — rule P.

(1,2) ③ $\sim r$ — rule T, R.D

③ ④ $\sim r \to s \wedge \sim t$ — rule P, CP.

(3,4) ⑤ $s \wedge \sim t$ — rule T, R.D.

⑥ $(s \wedge \sim t) \to (a \vee b)$ — rule P.

(5,6) ⑦ $a \vee b$ — rule T, R.D.

④ Show that $(p \to q) \wedge (r \to s)$, $(q \to t) \wedge (s \to u)$, $\sim(t \wedge u)$ and $p \to r \Rightarrow \sim p$.

Sol:-

$H_1 : (p \to q) \wedge (r \to s)$

$H_2 : (q \to t) \wedge (s \to u)$

$H_3 : \sim(t \wedge u)$

$H_4 : p \to r$

$\therefore \sim p$

① $(p \to q) \wedge (r \to s)$ — rule P.

(1) ② $p \to q$ — rule T, simplification

(1) ③ $r \to s$ — rule T, "

④ $(q \to t) \wedge (s \to u)$ — rule P

(4) ⑤ $q \to t$ — rule T, simplification

(4) ⑥ $s \to u$ — rule T "

(2,5) ⑦ $p \to t$ — rule T, (T.P).

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(3,6) 8 $r \to u$ — rule T, T.P

9 $p \to r$ — rule P.

(9,8) 10 $p \to u$ — rule T (T.P).

(4) 11 $\sim t \to \sim p$ — rule T, contrapositive.

(10) 12 $\sim u \to \sim p$ — rule T, "

(11,12) 13 $(\sim t \vee \sim u) \to \sim p$ — rule T, rule of proof by cases.

(13) 14 $\sim(t \wedge u) \to \sim p$ — rule T, Demorgan's

15 $\sim(t \wedge u)$ — rule P

(14,15) 16 $\sim p$ — rule T, R.D.

5 Show that $t \wedge s$ can be derived from the premises $p \to q$, $q \to \sim r$, r, $p \vee (t \wedge s)$.

sol:

$H_1: p \to q$

$H_2: q \to \sim r$

$H_3: r$

$H_4: p \vee (t \wedge s)$

$C: \therefore t \wedge s$.

1 $p \to q$ — rule P.

2 $q \to \sim r$ — rule P.

(1,2) 3 $p \to \sim r$. — rule T, R.D.

(3) 4 $r \to \sim p$ — rule T, contrapositive.

5 r — rule P.

(4,5) 6 $\sim p$. — rule T, R.D

7 $p \vee (t \wedge s)$ — rule P.

(7) (8) $\sim P \rightarrow (t \wedge s)$ — rule T, Implication.

(6,8) (9) $t \wedge s$ — rule T, R.D.

(6) Show that $P \rightarrow Q,\ Q \rightarrow \sim R,\ R,\ P \vee (J \wedge S) \Rightarrow J \wedge S$.

Solution:-

$H_1 : P \rightarrow Q$

$H_2 : Q \rightarrow \sim R$

$H_3 : R$.

$H_4 : P \vee (J \wedge S)$.

$\therefore C : J \wedge S$

(1) $P \rightarrow Q$ — rule P.

(2) $Q \rightarrow \sim R$ — rule P.

(1,2) (3) $P \rightarrow \sim R$ — rule T, T.P.

(3) (4) $R \rightarrow \sim P$ — rule T, contrapositive.

(5) R — rule P.

(4,5) (6) $\sim P$ — rule T, R.D.

(7) $P \vee (J \wedge S)$ — rule P.

(7) (8) $\sim P \rightarrow (J \wedge S)$ — rule T, implication.

(6,8) (9) $J \wedge S$ — rule T, R.D.

Rule CP: If we can derive S from R and set of premises then, we can derive $R \to S$ from the set of premises alone.

→ The general idea of this rule is that we may introduce a new premise R. Conditionally and use it in the conjunction with the original premises to derive a conclusion S, and then assert that the implication $R \to S$ follows from the original premises alone.

→ If 'S' is a valid premise or inference from the premises $P_1, P_2, P_3 \dots P_n$ and R. Then $R \to S$, is a valid inference from premises $P_1, P_2, P_3 \dots P_n$.

→ Rule 'CP' generally used if the conclusion is of the form $R \to S$. In such cases, R is taken as an additional premise and 'S' is derived from the given premises and R.

Ex: Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\sim R \vee P$ and Q.

Sol: we get $H_1: P \to (Q \to S)$

$H_2: \sim R \vee P$

$H_3: Q$

$\therefore C: R \to S$.

	(1)	R	— Rule P (additional premise)
	(2)	$\sim R \vee P$	— Rule P.
(2)	(3)	$R \to P$	— Rule T, implication
(1,3)	(4)	P	— Rule T, Modus Ponens.
	(5)	$P \to (Q \to S)$	— Rule P.
(4,5)	(6)	$(Q \to S)$	— Rule T, M.P.
	(7)	Q	— Rule P.
(6,7)	(8)	S	— Rule T, M.P
(1,8)	(9)	$R \to S$.	

Here we can derive 'S' from 'R' and set of premises.

then we derive $R \to S$ from the set of premises alone.

Ex: Derive $P \to (Q \to S)$ using CP-rule (if necessary) from the premises $P \to (q \to r)$ and $q \to (r \to s)$.

Sol: we shall assume 'P' as an additional premise. we will derive using P and the two given premises, $(q \to s)$.

Then by CP-rule $P \to (q \to s)$ is derived from the two given premises.

$H_1: P \to (q \to r)$

$H_2: q \to (r \to s)$

$\therefore P \to (q \to s)$.

① P — rule P (additional)

② $P \to (q \to r)$ — rule P

(1,2) ③ $q \to r$ — rule T, (Modus ponens)

(3) ④ $\sim q \vee r$ — rule T, (equivalence)

⑤ $q \to (r \to s)$ — rule P.

⑥ $\sim q \vee (r \to s)$ — rule T, implication

(4,6) ⑦ $\sim q \vee (r \wedge (r \to s))$ — rule T, distributive.

$bvc \quad bva$

$bv(a \wedge c)$

MP

(7) ⑧ $\sim q \vee s$ — rule T, modus ponens.

⑨ $q \to s$ — rule T, implication

⑩ $p \to (q \to s)$ — T, cp rule.

$\therefore$ $q \to s$ can be derived from the P and set of given premises.

$\therefore$ $p \to (q \to s)$ is a valid conclusion.

Ex: Show that $P \to S$ can be derived from the premises $\sim P \vee Q$, $\sim Q \vee R$, $R \to S$.

Sol: We include P as an additional premise and derive S.

$H_1 : \sim P \vee Q$

$H_2 : \sim Q \vee R$

$H_3 : R \to S$

$\therefore C : P \to S$

	(1)	$\sim P \vee Q$	— Rule P
(1)	(2)	$P \to Q$	— Rule T, Implication
	(3)	$\sim Q \vee R$	— Rule P.
(3)	(4)	$Q \to R$	— Rule T, Implication
(2,4)	(5)	$P \to R$	— Rule T, T.P.
	(6)	$R \to S$	— Rule P.
(5,6)	(7)	$P \to S$	— Rule T, T.P
	(8)	P	— Rule P (additional premise)
(7,8)	(9)	S	— Rule T, M.P.
(8,9)	(10)	$P \to S$	— Rule CP.

P

$P \to Q$

Q

$Q \to R$

R

$R \to S$

S

$P \to S$

Example:- Derive the following, using rule CP, if necessary.

(a) $\sim p \vee q$, ~~$\sim q \vee p$~~ ~~$(q \to s)$~~ $\Rightarrow p \to q$.

Sol:-

	(1)	p	— rule P (additional premise)
	(2)	$\sim p \vee q$	— rule P.
(2)	(3)	$p \to q$	— rule T, implication.
(1, 3)	(4)	q	— rule T, (R.D).
(1, 4)	(5)	$p \to q$	— rule CP.

(b) $p, p \to (q \to (r \wedge s)) \Rightarrow (q \to s)$.

Sol:-

	(1)	p	— rule P
	(2)	$p \to (q \to (r \wedge s))$	— rule P.
(1, 2)	(3)	$q \to (r \wedge s)$	— rule T, R.D.
	(4)	q	— rule P (additional premise)
(3, 4)	(5)	$r \wedge s$	— rule T (R.D)
(5)	(6)	s	— rule T, simplification
(4, 6)	(7)	$q \to s$	— rule CP.

Consistency of premise: A set of premises H_1, H_2, H_3 --- H_n are said to be ~~consistency of the~~ consistency if their conjunction $H_1 \wedge H_2 \wedge$ -- $\wedge H_m$ has the truth value 'T' in

$P \to Q$. — $Q \to P$

$\sim P \to \sim Q$

$\sim Q \to \sim P$