Consistency of premises.

A set of premises H_1, H_2, H_3 --- H_n are said to be consistency if their conjunction has the truth value 'T'. in $H_1 \wedge H_2 \wedge H_3$ --- H_n has the truth value 'T' in atleast one possible situation.

Example:- show that the following premises are consistent.

$p \vee q, \sim p$.

p	q	$p \vee q$	$\sim p$	$((p \vee q) \wedge \sim p)$
F	F	F	T	F
F	T	T	T	T
T	F	T	F	F
T	T	T	F	F

The above premises are consistency because $(p \vee q) \wedge \sim p$ is true only if p is false and q is true.

premises.

Inconsistency of premises:

A set of premises $H_1, H_2, H_3 \ldots H_n$ are said to be inconsistent if their conjunction $H_1 \wedge H_2 \ldots \wedge H_n$ has the truth value 'F' in every possible situation.

Example:

Show that the following premises are inconsistent.

$\sim p \wedge q,\ p$.

p	q	$\sim p$	$\sim p \wedge q$	$(\sim p \wedge q) \wedge p$
F	F	T	F	F
F	T	T	T	F
T	F	F	F	F
T	T	F	F	F

∴ The above premises are inconsistent because $(\sim p \wedge q) \wedge p$ is false in all possible situations.

Consistent of premises:

A set of premises $H_1, H_2, H_3 \ldots H_n$ is said to be consistent if their conjunction has the truth value T for some assignment of the truth values to the atomic variables appearing in $H_1, H_2, H_3 \ldots H_n$.

Inconsistent of premises

A set of premises (formulas) $H_1, H_2, H_3 \ldots H_n$ is said to be in consistent, if their conjunction implies a contradiction. i.e if for every assignment of the truth values to the atomic variables, at least one of the formulas $H_1, H_2 \ldots H_m$ is false, so that their conjunction is identically false. then the formulas $H_1, H_2, H_3 \ldots H_m$ are called inconsistent, that is, a set of formulas $H_1, H_2 \ldots H_m$ is called inconsistent if their conjunction implies a contradiction.

Note:- A necessary and sufficient condition for the implication that $H_1, H_2, \ldots H_n$ is a contradiction is that $r \wedge \neg r$.

Example: Show that $p \to q$, $p \to r$, $q \to \neg r$ and p are inconsistent.

Sol:- Given premises are.

$H_1 : p \to q$

$H_2 : p \to r$

$H_3 : q \to \neg r$

$H_4 : p$

$H_1 \wedge H_2 \wedge H_3 \wedge H_4$:. leads to a contradiction.

(1) $p \to q$ — rule P.

(2) $q \to \sim r$ — rule P.

(1,2) (3) $p \to \sim r$ — rule T, (Transitive property)

(4) p — rule P.

(3,4) (5) $\sim r$ — rule T, (Modus Ponens)

(6) $p \to r$ — rule P.

(6) (7) $\sim r \to \sim p$ — rule T, contrapositive.

(5,7) (8) $\sim p$ — rule T, Modus ponens.

(4,8) (9) $p \land \sim p$ — simplification, conjunction

But $p \land \sim p \Leftrightarrow F$

$\therefore$ the given premises are inconsistent.

Example: Show that the premises $a \to (b \to c)$, $d \to (b \land \sim c)$, $a \land d$ are inconsistent.

Sol:

(1) $a \land d$ — Rule P.

(1) (2) a — Rule T, simplification

(3) $a \to (b \to c)$ — Rule P.

(2,3) (4) $b \to c$ — Rule T, (R.D).

(1) (5) d — Rule T, simplification

(6) $d \to (b \land \sim c)$ — Rule P.

(5,6) (7) $b \land \sim c$. — Rule T, (R.D).

(4) (8) $\sim c \to \sim b$ — Rule T, Contrapositive.

(8) (9) $c \lor \sim b$ — Rule T, implication

(10) $\sim (b \land \sim c)$ — Rule T,

(7,10) (11) $(b \land \sim c) \land \sim (b \land \sim c)$ — Contradiction.

given premises are inconsistent.

$\sim c \to \sim b$.
$c \lor \sim b$.
$\sim (b \land \sim c)$.

B: He will go bankrupt

II method

① $a \wedge d$ — Rule p.

(1) ② a — Rule T } simplification.

(1) ③ d — Rule T

④ $a \to (b \to c)$ — Rule p.

(2, 4) ⑤ $b \to c$ — Rule T, (R.D).

⑥ $d \to (b \wedge \sim c)$ — Rule p.

(6) (7) $\sim(b \wedge \sim c) \to \sim d$ — Rule T, contrapositive.

(8) $(\sim b \vee c) \to \sim d$ — Rule T, demorgans

(5) ⑨ $\sim b \vee c$ — Rule T, implication

(8, 9) ⑩ $\sim d$ — Rule T, (R.D).

(3, 10) ⑪ $d \wedge \sim d$ — contradiction.

∴ the given set of premises are inconsistent.

Example: Show that the following set of premises are inconsistent.

→ If the contract is valid, then john is liable for penalty.

→ If john is liable for penalty, he will go bankrupt,

→ If the bank will loan him money, he will not go bankrupt.

as the matter of fact the contract is valid and the bank will loan him money.

Sol: we will indicate the given stmt as follows.

V: the contract is valid.

L: John is liable for penalty.

M: Bank will loan him money.

B: He will go bankrupt.

then the given premises are.

$V \to L,\ L \to B,\ M \to \sim B,\ V \wedge M$.

① $V \to L$ — Rule P

② $L \to B$ — Rule P.

(1,2) ③ $V \to B$ — Rule T, (Transitive property)

④ $V \wedge M$ — Rule P.

(4) ⑤ V — Rule T } simplification.

(4) ⑥ M — Rule T }

(3,5) ⑦ B — Rule T, (Modus ponens)

⑧ $M \to \sim B$ — Rule P.

(6,8) ⑨ $\sim B$ — Rule T, Modus ponens.

⑩ $B \wedge \sim B$ — Rule T, (Rule of contradiction)

∴ Thus the given set of premises leads to a contradiction and hence it is inconsistent.

Ex: Show that given set of premises are inconsistent.

① If Jack misses many classes through illness, then he fails high school.

② If Jack fails high school, then he is uneducated.

③ If Jack reads a lot of books, then he is not uneducated.

④ Jack misses many classes through illness and reads lot of books.

Sol:- E: Jack misses many classes.

S: Jack fails high school.

A: Jack reads a lot of books.

H: Jack is uneducated.

The premises are $E \to S$, $S \to H$, $A \to \neg H$, and $E \wedge A$.

(1) $E \to S$ — Rule P

(2) $S \to H$ — Rule P

(1,2) (3) $E \to H$ — Rule T, (Transitive property)

(4) $A \to \neg H$ — Rule P

(4) (5) $H \to \neg A$ — Rule T, Contrapositive.

(6) $E \to \neg A$ — Rule T, (T.P).

(6) (7) $\neg E \vee \neg A$ — Rule T, implication

(7) (8) $\neg(E \wedge A)$ — Rule T, Demorgans.

(9) $E \wedge A$ — Rule P.

(9,8) (10) $(E \wedge A) \wedge \neg(E \wedge A)$ — Rule T, Simplification.

∴ Thus, the given set of premises leads to a contradiction and hence it is inconsistent.

Ex:- Show that the following set of premises are inconsistent.

(1) If Rama gets his degree, then he will go for a job.

(2) If he goes for a job, he will get married soon.

(3) If he goes for higher study, he will not get married.

(4) Rama get his degree and goes for higher study.

Sol: Let the Stmt be symbolized as follows.

p : Rama gets his degree

q : He will go for a job.

r : He will get married soon.

s : He goes for higher study.

then we have to prove that.

$p \to q,\ q \to r,\ s \to \sim r,\ p \wedge s$ are inconsistent.

	(1)	$p \to q$	— rule P.
	(2)	$q \to r$	— rule P.
(1,2)	(3)	$p \to r$	— rule T, (Transitive property)
	(4)	$s \to \sim r$	— rule P.
(4)	(5)	$r \to \sim s$	— rule T, Contrapositive.
(3,5)	(6)	$p \to \sim s$	— rule T, T.P.
(6)	(7)	$\sim p \vee \sim s$	— rule T, implication
	(8)	$\sim (p \wedge s)$	— rule T, negation.
	(9)	$p \wedge s$	— rule P.
(8,9)	(10)	$(p \wedge s) \wedge \sim (p \wedge s)$	— rule T, Conjunction.
	(11)	F (Contradiction)	

∴ then the given set of premises leads to a contradiction.

Example: Test the validity of the following argument:

"Sonia is watching TV. If Sonia is watching TV, then she is not studying. If she is not studying, then her father will not buy her a scooty. Therefore, Sonia's father will not buy a scooty."

Sol:- Let us indicate the stmt as follows.

p: Sonia is watching TV

q: Sonia is studying

r: Sonia's father will buy a scooty.

Hence, the given argument is of the form

$p,\ p \to \sim q,\ \sim q \to \sim r \Rightarrow \sim r.$

$p \Rightarrow (p \vee q)$

(1) p — rule P

(2) $p \to \sim q$ — rule P.

(1,2) (3) $\sim q$ — rule T, R.D

(4) $\sim q \to \sim r$ — rule P

(3,4) (5) $\sim r$ — rule T, R.D.

Hence, it is a valid argument.

Example:- If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game". Show that these statements constitute a valid argument.

Sol:- Let us indicate the statements as follows.

p: there was a ball game.

q: Travelling was difficult.

r: They arrived on time.

Hence, the given premises are $p \to q$, $r \to \sim q$ and r.

the conclusion is $\sim p$.

thus, we need to prove that $p \to q,\ r \to \sim p,\ r \Rightarrow \sim p$.

(1) $r \to \sim q$ — rule P

(2) r — rule P

(1,2) (3) $\sim q$ — rule T, ~~(1),(2)~~ (R.D)

(4) $p \to q$ — rule P

(4) (5) $\sim q \to \sim p$ — rule T, contrapositive

(3,5) (6) $\sim p$ — rule T, (R.D)

Hence, the given statements constitute a valid argument.

Example:- By using the method of derivation, show that the following statements constitute a valid arguments: "If A works hard, then either B or C will enjoy. If B enjoys then A will not work hard. If D enjoys then 'C' will not enjoy. Therefore, if A works hard, D will not enjoy.

Sol:- let us indicate the statements as follows.

p: A works hard

q: B will enjoy

r: C will enjoy

s: D will enjoy

Given premises are $p \to (q \vee r)$, $q \to \sim p$, $s \to \sim r$.

The conclusion is $p \to \sim s$.

Thus, we need to prove that $p \to (q \vee r)$, $q \to \sim p$, $s \to \sim r$

$\Rightarrow (p \to \sim s)$.

① p — rule P (additional premise)

② $p \to (q \vee r)$ — rule P

(1,2) ③ $q \vee r$ — rule T, (R.D)

(3) ④ $\sim q \to r$ — rule T, implication.

⑤ $\sim r \to q$ — rule T, contrapositive.

⑥ $q \to \sim p$ — rule P.

(5,6) ⑦ $\sim r \to \sim p$ — rule T, (contrapositive) (T.P)

(7) ⑧ $p \to r$ — rule T, (contrapositive)

⑨ $s \to \sim r$ — rule P.

(9) ⑩ $r \to \sim s$ — rule T, (contrapositive)

(8,10) ⑪ $p \to \sim s$ — rule T, (T.P).

⑫

∴ Hence, the statements constitute a valid argument.

Indirect method of proof:

The method of using the rule of conditional proof and the notation of an inconsistent set of premises is called the Indirect method of proof (or) proof by contradiction.

The Techniques of indirect method of proof is as follows;

① Introduce the negation of the desired conclusion as a new premise.

that is, assume the conclusion 'C' is false and consider $\sim C$ as an additional premise or (new premise).

(2) From the additional or new premise, together with the given premise, derive a contradiction.

(4) that is, if the new set of premises is inconsistent then they imply a contradiction.

$\therefore$ 'C' is true whenever $H_1 \wedge H_2 \wedge H_3 \cdots H_m$ is true. R

(3) Assert the desired conclusion as a logical inference from the premises.

Thus 'C' follows logically from the premises $H_1, H_2 \cdots H_m$.

<u>Example:-</u> Using indirect method of proof, derive $P \to \sim S$ from the $P \to Q \vee R$, $Q \to \sim P$, $S \to \sim R$, and P.

<u>Sol:-</u> The desired result is $P \to \sim S \Rightarrow \sim P \vee \sim S$
$\Rightarrow \sim(P \wedge S)$.

It's negation is $P \wedge S$. We include $P \wedge S$ is an additional premise.

	(1)	$P \to Q \vee R$	rule P.
	(2)	P	rule P.
(1,2)	(3)	$Q \vee R$	rule T, (R.D).
	(4)	$S \to \sim R$	rule P.
	(5)	$P \wedge S$	rule P (additional premise)
(5)	(6)	S	rule T, simplification.
(4,6)	(7)	$\sim R$	rule T, (Modus ponens)
(3,7)	(8)	Q	rule T (disjunctive syllogism)
	(9)	$Q \to \sim P$	rule P
(8,9)	(10)	$\sim P$	rule T, R.D
(2,10)	(11)	$P \wedge \sim P$	rule T, contradiction.

(4, 3) ⑤ $\sim P$ — rule T, (modus tolens)

⑥ $P \vee R$ — rule P.

(5, 6) ⑦ R — rule T, (disjunctive syllosim)

⑧ $R \wedge \sim R$ — rule T, contradiction.

Thus we get a contradiction.

$\therefore$ we get $P \to Q, Q \to R, P \vee R \Rightarrow R$.

but, the other premise $\sim(P \wedge R)$ will not give a contradiction with R.

<u>Example</u>:- Show that $\sim(P \wedge Q)$ follows from $\sim P \wedge \sim Q$.

<u>Sol</u>:- To prove the above statement, we will introduce the negation of the conclusion i.e. $\sim(\sim(P \wedge Q))$ as an additional premise. Then, we will show that this leads to a contradiction.

① $\sim(\sim(P \wedge Q))$ — rule P (assumed premise)

(1) ② $P \wedge Q$ — rule T, law of double negation.

(2) ③ P — rule T, simplification

4\. $\sim P \wedge \sim Q$ ④ $\sim P \wedge \sim Q$ — rule P

5\. $\sim Q$ (4) ⑤ $\sim Q$ — rule T, simplification

6\. Q (2) ⑥ Q — P

7\. $Q \wedge \sim Q$ (6, 5) ⑥ $Q \wedge \sim Q$ — rule T, contradiction.

Thus $P \wedge \sim P \Rightarrow F$.

Hence, our assumption is wrong.

Thus, $\sim(P \wedge Q)$ follows from $\sim P \wedge \sim Q$.

Example: Show that the following using the indirect method.

(a) $(r \to \sim q)$, $r \vee s$, $s \to \sim q$, $p \to q \Rightarrow \sim p$.

Sol: To prove the above statement, we will introduce the negation of conclusion.

i.e $\sim(\sim p)$, as an additional premise, and given set of premises leads to a contradiction.

	(1)	$p \to q$	— rule P.
	(2)	$\sim(\sim p)$	— rule P (additional premise)
(2)	(3)	p	— rule T, law of double negation.
(1,3)	(4)	q	— rule T, (R.D).
	(5)	$r \to \sim q$	— rule P.
(5)	(6)	$q \to \sim r$	— rule T, contrapositive.
(4,6)	(7)	$\sim r$	— rule T (R.D).
	(8)	$r \vee s$	— rule P
(8)	(9)	$\sim r \to s$	— rule T, implication
(6,9)	(10)	$q \to s$	— rule T, T.P.
	(11)	$s \to \sim q$	— rule P.
(10,11)	(12)	$q \to \sim q$	— rule T, (T.P).
(4,12)	(13)	$\sim q$	— rule T, (R.D).
(4,13)	(14)	$q \wedge \sim q$	— rule T, contradiction.

contradiction.

$\therefore$ $\sim p$ follows from, $(r \to \sim q)$, $r \vee s$, $s \to \sim q$, $p \to q$.

Example: show that the following by using indirect proof (method) $S \to \sim Q$, $S \vee R$, $\sim R$, $\sim R \rightleftarrows Q \Rightarrow \sim P$.

Sol: To prove the above statement, we will introduce the negation of the conclusion.

i.e $\sim(\sim P) = P$ additional premise + given set of premises are leads to a contradiction.

(1) $\sim R \rightleftarrows Q$ — rule P.

(1) (2) $(\sim R \to Q) \wedge (Q \to \sim R)$ — rule T (double implies)

(2) (3) $\sim R \to Q$ — rule T, simplification

(4) $\sim R$ — rule P

(3,4) (5) Q — rule T, (R.D).

(6) $S \to \sim Q$ — rule P.

(7) $S \vee R$ — rule P

(8) $\sim S \to R$ — rule T, implication

(9) $\sim R \to S$ — rule T, contrapositive.

(9,6) (10) $\sim R \to \sim Q$ — rule T, T.P.

(2) (11) $Q \to \sim R$ — rule T, simplification

(12) $Q \to \sim Q$ — rule T, T.P.

(5,12) (13) $\sim Q$ — R.D.

(5,13) (14) $Q \wedge \sim Q$ — Rule T, contradiction.

$\therefore$ P will not give the contradiction with $\sim P$.

Example:- Show by indirect proof

(a) $P \to Q, R \to S, P \vee R \Rightarrow Q \vee S$.

Sol: prove that above statement, we include a negation of the conclusion. i.e,

$\sim(Q \vee S)$. and given set of premises leads to a contradiction.

(1) $\sim(Q \vee S)$ — Rule P (additional premise)

(1) (2) $\sim Q \wedge \sim S$ — Rule T

(2) (3) $\sim Q$ — Rule T, simplification.

(4) $P \to Q$ — rule P

(4) (5) $\sim Q \to \sim P$ — rule T, Contrapositive.

(3, 5) (6) $\sim P$ — rule T, (R.D)

(7) $R \to S$ — rule P.

(7) (8) $\sim S \to \sim R$ — rule T, (contrapositive)

(2) (9) $\sim S$ — rule T, simplification

(8, 9) (10) $\sim R$. — rule T, (R.D)

(11) $P \vee R$ — rule P.

(11) (12) $\sim P \to R$ — rule T,

(6, 12) (13) R — rule T, (R.D).

(13, 10) (14) $R \wedge \sim R$ — Rule T, Contradiction,

$\therefore$ It is valid argument.

(b) $E \to S, S \to H, A \to \sim H \Rightarrow \sim(E \wedge A)$.

Sol:- We prove that above statement, we include a ~~conclusion of~~ negation of conclusion. The negation of conclusion and given set of premises are leads to a contradiction.

(1) $E \to S$ — Rule P

(2) $S \to H$ — Rule P.

(1,2) (3) $E \to H$ — Rule T, (T.P)

(3) (4) $\sim H \to \sim E$ — Rule T, contrapositive

(5) $A \to \sim H$ — Rule P.

(4,5) (6) $A \to \sim E$ — Rule T, T.P

(7) $\sim A \vee \sim E$ — Rule T, implication.

(7) (8) $\sim(E \wedge A)$ — Rule T, Demorgan's

(9) $E \wedge A$ — Rule P, additional.

(8,9) (10) $(E \wedge A) \wedge \sim(E \wedge A)$ — Rule T, Contradiction.

(c) If $P \to (Q \wedge R)$, $(Q \vee S) \to T$ and $P \vee S$ then T.

Sol:- We prove that.

$P \to (Q \wedge R), (Q \vee S) \to T, P \vee S \Rightarrow T$.

The negation of the conclusion $(\sim T)$ and given set of premises are leads to a contradiction.

(1) $Q \vee S \to T$ — Rule P.

(1) (2) $\sim T \to \sim(Q \vee S)$ — Rule T, Contrapositive.

(3) $\sim T$ — Rule P, additional premise.

(2,3) (4) $\sim(Q \vee S)$ — Rule T, (R·D).

(5) $P \vee S$ — Rule P.

(5) (6) $\sim P \to S$ — Rule T, implication.

(7) $\sim S \to P$ — Rule T, Contrapositive.

(8) $P \to (Q \wedge R)$ — Rule P.

(7,8) (9) $\sim S \to (Q \wedge R)$ — Rule T, (T·P).

(4) (10) $\sim Q \wedge \sim S$ — Rule T, Demorgans.

(11) $\sim S$. — Rule T, simplification.

(9,11) (12) $Q \wedge R$. — Rule T, (R·D).

(13) Q — Rule T, simplification

(10) (14) $\sim Q$ — " "

(15) $Q \wedge \sim Q$ — Rule T, Contradiction.

Predicate Calculus

Predicate logic:

Predicate:- predicate refers to the property of the Subject.

Predicate logic:-

The logic based on the analysis of predicates in any statements is called predicate logic.

For ex:- Consider two statements.

(1) Smitha is a beautiful.

(2) mythily is a beautiful.

→ If we express these two statements by symbols we require two different symbols to denote them.

→ The two symbols do not express the common fetures of these two statements.

→ Here the part "is a beautiful" is called predicate.

Smitha & mythily are the subjects.

→ Capital letters are denoted by predicates.

→ Small letters are denoted by subjects.

Ex:- B - is a beautiful.

s - Smitha.

m - mythily.

B(s), B(m) → mythily is a beautiful.

↓

Smitha is a beautiful.

The additional premise $P \wedge S$ and the given premises together leads to a contradiction.

So $\sim(P \wedge S)$ is derived from $P \to Q \vee R$, $Q \to \sim P$, $S \to \sim R$, P and $P \wedge S$.

Example:- prove by indirect method that $\sim Q, P \to Q, P \vee R \Rightarrow R$

Sol:- The desired result is R.

Include its negation as a new premise $\sim R$.

(1) $P \vee R$ — rule P.

(2) $\sim R$ — rule P (additional premise)

(1,2) (3) P — Rule T (disjunctive syllogism).

(4) $P \to Q$ — Rule P.

(3,4) (5) Q — Rule T, (R.D).

(6) $\sim Q$ — Rule P.

(5,6) (7) $Q \wedge \sim Q$ — rule T, contradiction.

The new premise, together with the given premise leads to a contradiction, thus $\sim Q, P \to Q, \sim R \Rightarrow R$.

Example:- By indirect proof, show that

$P \to Q, Q \to R, \cancel{\sim(P \wedge R)}, P \vee R \Rightarrow R$.

Sol:- The desired results is R, include its negation $\sim R$ is additional premise.

(1) $Q \to R$ — Rule P.

(2) $\sim R$ — Rule P (additional premise)

(1,2) (3) $\sim Q$ — Rule T (Modus Tollens)

(4) $P \to Q$ — Rule P.