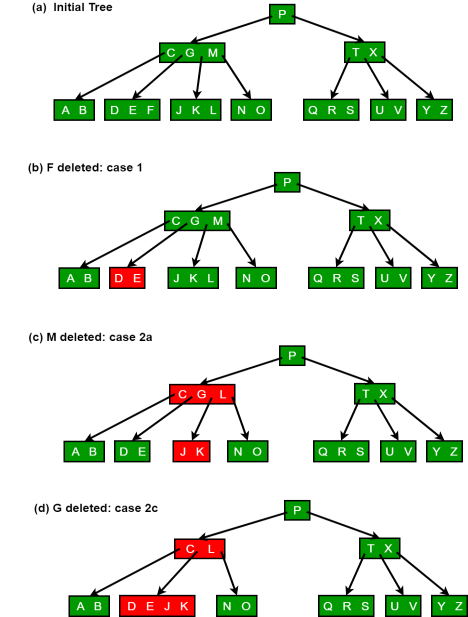
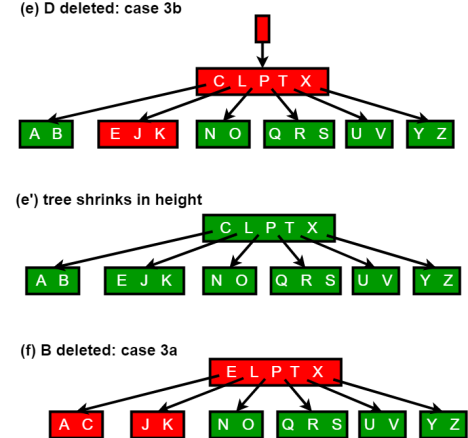
B-tree deletion





**2-3 trees**

In binary search trees we have seen the average-case time for operationslikesearch/insert/delete is O(log N) and the worst-case time is O(N) whereNisthenumber of nodes in the tree.

Like other Trees include AVL trees, Red Black Tree, B tree, **2-3 Tree isalsoaheight balanced tree**.

The time complexity of search/insert/delete is O(log N) .

A 2-3 tree is a *B-tree of order 3*.

**Properties of 2-3 tree:**

∙ Nodes with two children are called 2-nodes. The 2-nodes have onedatavalue and two children

∙ Nodes with three children are called 3-nodes. The 3-nodes have twodatavalues and three children.

∙ Data is stored in sorted order.

∙ It is a balanced tree.

∙ All the leaf nodes are at same level.

∙ Each node can either be leaf, 2 node, or 3 node.

∙ Always insertion is done at leaf.

**Search:** To search a key **K** in given 2-3 tree **T**, we follow the followingprocedure:

Base cases:

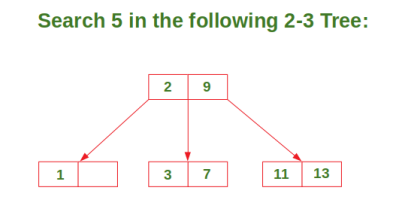
1. If **T** is empty, return False (key cannot be found in the tree). 2. If current node contains data value which is equal to **K**, return True. 3. If we reach the leaf-node and it doesn’t contain the required key value**K**, return False.

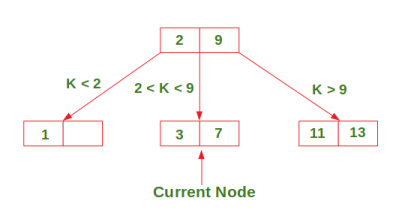
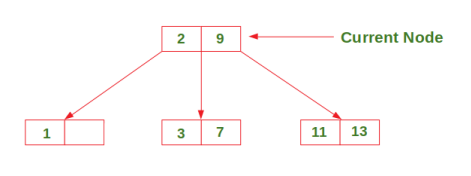
Recursive Calls:

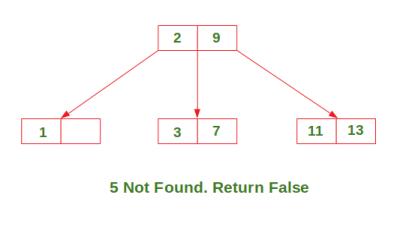
1. If **K** < currentNode.leftVal, we explore the left subtree of the current node. 2. Else if currentNode.leftVal < **K** < currentNode.rightVal, we explorethemiddlesubtree of the current node.

3. Else if **K** > currentNode.rightVal, we explore the right subtree of thecurrent node.

Consider the following example:

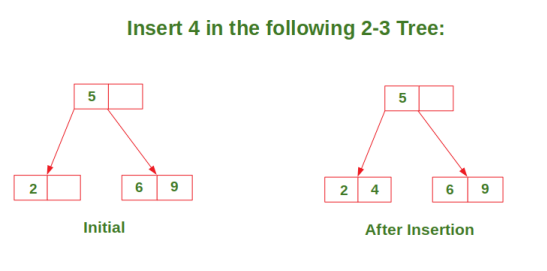




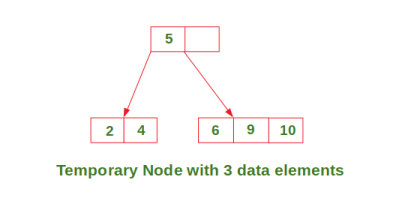
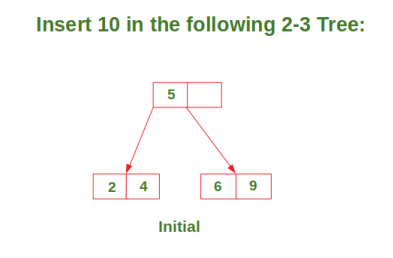


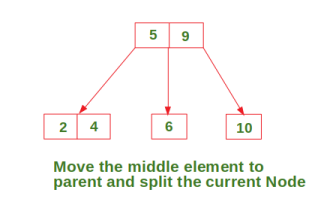
**Insertion:** There are 3 possible cases in insertion which have been discussedbelow:

∙ **Case 1:** Insert in a node with only one data element

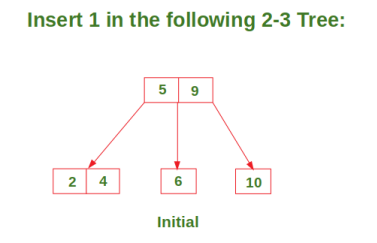


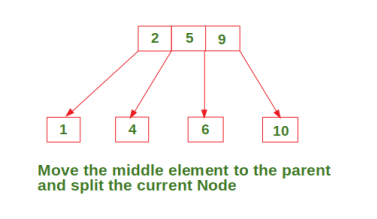
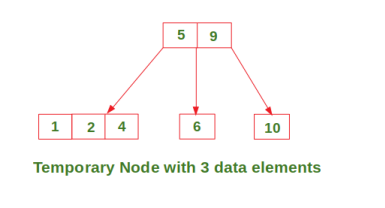
∙ **Case 2:** Insert in a node with two data elements whose parent containsonlyone data element.

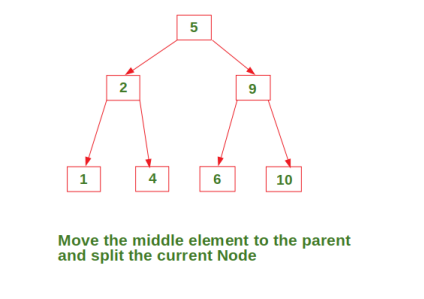




∙ **Case 3:** Insert in a node with two data elements whose parent alsocontainstwo data elements.







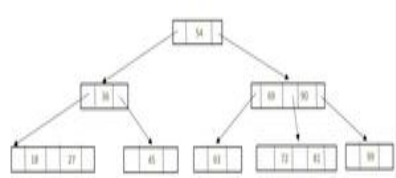
**In Deletion Process** for a specific value:

∙ To delete a value, it is replaced by its in-order successor and thenremoved. ∙ If a node is left with less than one data value then two nodes must bemerged together.

∙ If a node becomes empty after deleting a value, it is then merged withanother node.

To Understand the deletion process-

Consider the 2-3 tree given below



*Given 2-3 Tree*

*delete the following values from it: 69,72, 99, 81*.

**To delete 69**, swap it with its in-order successor, that is, 72. 69 nowcomesinthe leaf node. Remove the value 69 from the leaf node. *After deletion 69*

**To delete 72**, 72 is an internal node. To delete this value swap 72 withitsin- order successor 81 so that 72 now becomes a leaf node. Remove thevalue72from the leaf node.



*After deletion 72*

Now there is a leaf node that has less than 1 data value thereby violatingtheproperty of a 2-3 tree. So the node must be merged.

To merge the node, pull down the lowest data value in the parent’s nodeandmerge it with its left sibling.



*Rebalancing to Satisfy 23 Tree property*

**To delete 99**, 99 is present in a leaf node, so the data value can be easilyremoved.



*After deletion 99*

Now there is a leaf node that has less than 1 data value, thereby violatingtheproperty of a 2-3 tree.

So the node must be merged. To merge the node, pull down the lowest datavalue in the parent’s node and merge it with its left sibling.



*Rebalancing to Satisfy 2-3 Tree Property*

**To delete 81**, 81 is an internal node. To delete this value swap 81 withitsin- order successor 90 so that 81 now becomes a leaf node. Remove thevalue81from the leaf node.



*After deletion 81*

Now there is a leaf node that has less than 1 data value, thereby violatingtheproperty of a 2-3 tree. So the node must be merged. To merge the node, pull down the lowest data value in the parent’s node and merge it with its left sibling.



*Rebalancing to Satisfy 2-3 Tree property*

As internal node cannot be empty. So now pull down the lowest data valuefromthe parent’s node and merge the empty node with its left sibling



*Rebalancing to Satisfy 2-3 Tree Property*