

1. Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex t at a distance four from the root. If t is the n -th vertex in this BFS traversal, then the maximum possible value of n is _____.

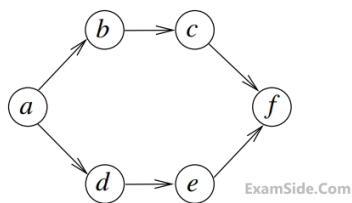
A 29

B 30

C 31

D 32

2.



Consider the following directed graph:

The number of different topological orderings of the vertices of the graph is _____.

A 5

B 6

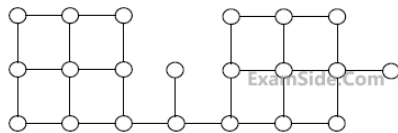
C 7

D 8

3. Let G be a graph with n vertices and m edges. What is the tightest upper bound on the running time of Depth First Search on G , when G is represented as an adjacency matrix?

- | | | | |
|----------------------------|---------------|----------------------------|----------|
| <input type="checkbox"/> A | $O(n \log n)$ | <input type="checkbox"/> B | $O(n!)$ |
| <input type="checkbox"/> C | $O(m^2)$ | <input type="checkbox"/> D | $O(n^2)$ |

4.



Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is _____.

- | | | | |
|----------------------------|----|----------------------------|----|
| <input type="checkbox"/> A | 17 | <input type="checkbox"/> B | 18 |
| <input type="checkbox"/> C | 19 | <input type="checkbox"/> D | 20 |

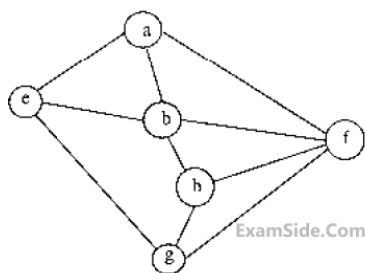
5. Consider the tree arcs of a BFS traversal from a source node **W** in an unweighted, connected, undirected graph. The tree **T** formed by the tree arcs is a data structure for computing

- | | |
|--|---|
| <p>A longest paths from the source node to all other nodes in the graph</p> | <p>B paths from all nodes to the source node in the graph</p> |
| <p>C random paths from the source node to all other nodes in the graph</p> | <p>D shortest paths from the source node to all other nodes in the graph</p> |

6. In a depth-first traversal of a graph G with n vertices, k edges are marked as tree edges. The number of connected components in G is

- | | |
|--|--|
| <p>A $n - k + 1$</p> | <p>B n / k</p> |
| <p>C $n - k$</p> | <p>D $n + k + 1$</p> |

7.



Consider the following graph among the following sequences

- I. a b e g h f
- II. a b f e h g
- III. a b f h g e
- IV. a f g h b e

- | | |
|------------------------------|--------------------------------|
| <p>A I. II.</p> | <p>B I. III. IV.</p> |
| <p>C II. III. IV.</p> | <p>D I. II. III. IV</p> |

8. Let G be a graph with $100!$ vertices, with each vertex labelled by a distinct permutation of the numbers $1, 2, \dots, 100$. There is an edge between vertices U and V if and only if the label of U can be obtained by swapping two adjacent numbers in the label of V . Let y denote the degree of a vertex in G , and z denote the number of connected components in G . Then, $y+10z=$ ____.

- | | | | |
|----------------------------|-----|----------------------------|-----|
| <input type="checkbox"/> A | 110 | <input type="checkbox"/> B | 109 |
| <input type="checkbox"/> C | 108 | <input type="checkbox"/> D | 107 |

9. In an adjacency list representation of an undirected simple graph $G=(V,E)$, each edge (u,v) has two adjacency list entries: $[v]$ in the adjacency list of u , and $[u]$ in the adjacency list of v . These are called twins of each other. A twin pointer is a pointer from an adjacency list entry to its twin. If $|E|=m$ and $|V|=n$, and the memory size is not a constraint, what is the time complexity of the most efficient algorithm to set the twin pointer in each entry in each adjacency list?

- | | | | |
|----------------------------|----------------|----------------------------|----------|
| <input type="checkbox"/> A | $O(m \cdot n)$ | <input type="checkbox"/> B | $O(m^2)$ |
| <input type="checkbox"/> C | $O(m+n)$ | <input type="checkbox"/> D | $O(n^2)$ |

10. Let $G = (V, E)$ be a simple undirected graph, and s be a particular vertex in it called the source. For $x \in V$, let $d(x)$ denote the shortest distance in G from s to x . A breadth first search (BFS) is performed starting at s . Let T be the resultant BFS tree. If (u, v) is an edge of G that is not in T , then which one of the following CANNOT be the value of $d(u) - d(v)$?

- | | | | |
|----------------------------|----|----------------------------|---|
| <input type="checkbox"/> A | -1 | <input type="checkbox"/> B | 0 |
| <input type="checkbox"/> C | 1 | <input type="checkbox"/> D | 2 |

11. Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes _____.

- | | | | |
|----------------------------|------|----------------------------|------|
| <input type="checkbox"/> A | 1000 | <input type="checkbox"/> B | 995 |
| <input type="checkbox"/> C | 1005 | <input type="checkbox"/> D | 1010 |

12. Let G be a weighted graph with edge weights greater than one and G' be the graph constructed by squaring the weights of edges in G . Let T and T' be the minimum spanning trees of G and G' respectively, with total weights t and t' . Which of the following statements is TRUE?

- | | | | |
|----------------------------|-----------------------|----------------------------|--------------------|
| <input type="checkbox"/> A | $T' = T, t' = t^2$ | <input type="checkbox"/> B | $T' = T, t' < t^2$ |
| <input type="checkbox"/> C | $T' \neq T, t' = t^2$ | | |

13.

Consider a complete undirected graph with vertex set $\{0,1,2,3,4\}$. Entry W_{ij} in the matrix W below is the weight of the edge $\{i, j\}$

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T ?

What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges?

☐ A 7,8

☐ B 8,9

☐ C 9,10

☐ D 10,8

14. What is the largest integer m such that every simple connected graph with n vertices and n edges contains at least m different spanning trees?

☐ A 1

☐ B 2

☐ C 3

☐ D n

15. Consider the depth-first-search of an undirected graph with 3 vertices P, Q, and R. Let discovery time $d(u)$ represent the time instant when the vertex u is first visited, and finish time $f(u)$ represent the time instant when the vertex u is last visited. Given that

$$d(P) = 5 \text{ units} \quad f(P) = 12 \text{ units}$$

$$d(Q) = 6 \text{ units} \quad f(Q) = 10 \text{ units}$$

$$d(R) = 14 \text{ unit} \quad f(R) = 18 \text{ units}$$

Which one of the following statements is TRUE about the graph

- | | | | |
|----------------------------|-----------------------|----------------------------|---------------------------|
| <input type="checkbox"/> A | P and Q are connected | <input type="checkbox"/> B | R and Q are connected |
| <input type="checkbox"/> C | P and R are connected | <input type="checkbox"/> D | P and Q are not connected |

Answer Key

1. c

2. b

3. d

4. c

5. d

6. c

7. b

8. b

9. c

10. d

11. b

12. b

13. d

14. c

15. a