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(AUTONOMOUS)

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DISCRETE MATHEMATICS AND GRAPH THEORY

Duration: 3 Hours **Max. Marks** : 70

PART-A

1.a) Define Proposition. [CO1-L1] [2M]

Ans: A proposition is a declarative sentence that can be either true or false.

1.b) What is Difference between CNF and PCNF? [CO1-L1][2M]

Ans:

CNF	PCNF
CNF- Conjunctive Normal Form	**PCNF-** Principal Conjunctive Normal Form
An equivalent formula consisting of conjunctions (product) of elementary sums.	An equivalent formula consisting of conjunctions(product) of maxterms only is called the principle conjunctive normal form of the formula
Ex: $(\sim P \vee Q) \wedge (Q \vee R)$	**Ex:** $(P \vee \sim Q \vee \sim R) \wedge (P \vee \sim Q \vee R) \wedge (\sim P \vee \sim Q \vee \sim R)$.

1.c) Let Q(x) be the statement " x<2." What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers? [CO2-L1][2M]

Ans: Q(x) is not true for every real number x, because, for instance, Q(3) **"$3 \not< 2$ "** is false. That is, x = 3 is a counterexample for the statement $\forall x\, Q(x)$.

Thus **$\forall x\, Q(x)$ is false.**

1.d) Explain existential quantifier. [CO2-L1][2M]

Ans: an existential quantifier is a logical constant that indicates **"there exists"**, **"there is at least one"**, or **"for some"**.

It is denoted by the symbol : $\exists$

1 e) Define non-homogeneous recurrence relation of order three. [CO3-L2][2M]

Ans: A non-homogeneous linear recurrence relation is an equation that relates a sequence of numbers where each term is a linear combination of previous terms, plus a function of the index (particular solution).

$a_n + a_{n-1} + a_{n-2} + a_{n-3} = f(n)$.

where $f(n) \neq 0$.

(OR)

$$a_n = a_n^{(h)} + a_n^{(p)}.$$

1.f) **Solve $a_n + 4a_{n-1} = 2$. [CO3-L2][2M]**

Sol:- Given Recurrence relation $a_n + 4a_{n-1} = 2$. — ①

General solution of ①.

$$a_n = a_n^{(h)} + d_n^{(p)}$$

$a_n^{(h)} \Rightarrow a_n + 4a_{n-1} = 0$ — ②

Characteristic equation of ②

$r + 4 = 0 \Rightarrow r = -4$.

Homogeneous solution of ② is.

$a_n^{(h)} = c_1(-4)^n$ — ③

Particular solution of ① is

$a_n^{(p)} = d$.

$d + 4d = 2$.

$5d = 2 \Rightarrow d = 2/5$. — ④

$a_n^{(p)} = 2/5$. — ④

Final solution is

$$\boxed{a_n = c_1(-4)^n + 2/5}$$

1.g) Write Warshalls Algorithm. [CO4-L2][2M]

Given the adjacency matrix A of a simple digraph, then the following steps produce the path matrix $P(\text{or} A^+)$:

Step 1: $P^{[0]} = A$

Step 2: $K = 1$

Step 3: $i = 1$

Step 4: $p_{ij}^{[K]} = p_{ij}^{[K-1]} \vee (p_{ik}^{[K-1]} \wedge p_{kj}^{[K-1]}) \forall j = 1$ to n

Step 5: $i = i + 1$. If $i \leq n$, go to step4

Step 6: $K = K + 1$ If $K \leq n$, go to step3; otherwise, stop.

1.h) Define a Directed Graph. . [CO4-L1][2M]

Ans: A directed graph, also known as a digraph, is a graph where the edges have a direction, usually indicated by an arrow.

1.i) Define minimal spanning tree. [CO4-L1][2M]

Ans: A minimum spanning tree (MST) is defined as a spanning tree that has the minimum weight among all the possible spanning trees.

1.j)Define Hamiltonian Graph. [CO4-L1][2M]

Ans: A Hamiltonian graph is a graph that contains a Hamiltonian cycle/circuit. A Hamiltonian cycle (or Hamiltonian circuit) is a cycle in a graph that visits every vertex exactly once except starting vertex and ending vertex.

(UNIT-I) PART-B

2(a) Show that the proposition $(p \vee \sim q) \wedge (\sim p \vee \sim q) \vee q$ is a tautology.

Ans:

			①		②	③	
p	q	$\sim q$	$p \vee \sim q$	$\sim p$	$\sim p \vee \sim q$	① $\wedge$ ②	③ $\vee q$
F	F	T	T	T	T	T	T
F	T	F	F	T	T	F	T
T	F	T	T	F	T	T	T
T	T	F	T	F	F	F	T

$\therefore$ It is a tautology.

2(b): For any three propositions p, q, r, prove that

$$[(p \vee q) \to r] \Leftrightarrow [(p \to q) \wedge (p \to r)]$$

① L.H.S ② R.H.S.

Ans:

p	q	r	$p \vee q$	$[(p \vee q) \to r]$	$p \to q$	$p \to r$	$[(p \to q) \wedge (p \to r)]$
F	F	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	T	F	T	F	T	T	T
F	T	T	T	T	T	T	T
T	F	F	T	F	F	F	F
T	F	T	T	T	F	T	F
T	T	F	T	F	T	F	F
T	T	T	T	T	T	T	T

$\therefore$ L.H.S $\neq$ R.H.S.

$$[(p \vee q) \to r] \neq [(p \to q) \wedge (p \to r)]$$

3(a) construct the truth (OR) table of the compound proposition
$$(p \vee \sim q) \rightarrow (p \wedge q).$$

Ans:-

p	q	$\sim q$	$p \vee \sim q$	$p \wedge q$	$(p \vee \sim q) \rightarrow (p \wedge q)$.
F	F	T	T	F	F
F	T	F	F	F	T
T	F	T	T	F	F
T	T	F	T	T	T

∴ The last column gives the result of compound proposition.

3(b) obtain the PDNF of the following.
$$P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$$

Ans:- PDNF - principal Disjunctive Normal form.

$P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

$P \rightarrow ((\sim P \vee Q) \wedge (Q \wedge P))$

$P \rightarrow [(\sim P \wedge (Q \wedge P)) \vee (Q \wedge (Q \wedge P))]$

$P \rightarrow ((P \wedge \sim P) \wedge Q) \vee ((Q \wedge Q) \wedge P))$ ∵ $P \wedge \sim P = F$ ∵ $Q \wedge Q = Q$.

$P \rightarrow [(F \wedge Q) \vee (P \wedge Q)]$ ∵ $F \wedge Q = F$.

$P \rightarrow [F \vee (P \wedge Q)]$ $F \vee P = P$

$P \rightarrow (P \wedge Q)$

$\sim P \vee (P \wedge Q)$.

$(\sim P \wedge T) \vee (P \wedge Q)$

$[\sim P \wedge (Q \vee \sim Q)] \vee (P \wedge Q)$

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$.

$\therefore$ Required PDNF form is.

PDNF / $\boxed{(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)}$

DNF : $\boxed{\sim P \vee (P \wedge Q)}$

UNIT-II

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4 (a) Ans: All Lions are fierce

Some lions do not drink coffee

Some fierce creatures do not drink coffee.

$P(x)$: x is a lion

$Q(x)$: x is a fierce.

$R(x)$: x drinks coffee.

Ans:

① All Lions are fierce: $\boxed{\forall(x)(P(x) \rightarrow Q(x))}$

② Some lions do not drink coffee:

$\boxed{\exists(x)(P(x) \wedge \sim R(x))}$.

③ Some fierce creatures do not drink coffee:

$\boxed{\exists(x)(Q(x) \wedge \sim R(x))}$

4 (b): Assume that "for all positive integers n", if n is greater than 4, then n^2 is less than 2^n" is true. Use universal modus ponens to show that $100^2 < 2100$. → consider this stmt as $\boxed{100^2 < 2^{100}}$

Ans:- let $P(n)$ denote "$n > 4$"

$Q(n)$ denote "$n^2 < 2^n$"

The statement "for all positive integers n. if 'n' is greater than 4, the n^2 less than 2^n".

It can be represented by

$$\boxed{\forall(n)\ P(n) \to Q(n)}$$

where the domain consists of all positive integers.

we are assuming that $\forall n\,(P(n) \to Q(n))$ is true.

Note that $P(100)$ is true because $\boxed{100 > 4}$.

It follows by universal modus ponens that $Q(100)$ is true, namely that

$$\boxed{100^2 < 2^{100}}$$ — True.

5(a) :- (OR)

H_1: A Student in this class has not read the book.

H_2: Everyone in this class passed the first exam.

C: Someone who passed the first exam has not read the book.

Ans: $C(x)$: x is in this class.

$B(x)$: x has read the book.

$P(x)$: x passed the first exam.

The given premises are :

$$H_1: \exists(x)\,(C(x) \wedge \sim B(x)).$$

$$H_2: \forall(x)\,(C(x) \rightarrow P(x)).$$

$$C: \exists(x)\,(P(x) \wedge \sim B(x))$$

	1.	$\exists(x)\,(C(x) \wedge \sim B(x))$	— rule P.
{1}	2.	$C(a) \wedge \sim B(a)$	— rule EG
{2}	3.	$C(a)$	— rule T, Simplification ~~conjunction~~.
	4.	$\forall(x)\,(C(x) \rightarrow P(x))$	— rule P.
{4}	5.	$C(a) \rightarrow P(a)$	— rule UG
{3,5}	6.	$P(a)$	— rule T, MP. Modus ponens.
{2}	7.	$\sim B(a)$	— rule T, ~~conjunction~~. Simplification,
{6,7}	8.	$P(a) \wedge \sim B(a)$	— rule T, conjunction
{8}	9.	$\exists(x)\,(P(x) \wedge \sim B(x))$	— rule T, ES.

$\therefore$ It is a valid conclusion.

5 (b) Use contrapositive show that if x and yare integers and both xy and x+y are even then both x and y are even.
Ans:

Proof of the Contrapositive

Assume that at least one of x or y is odd.

We need to show that either $x + y$ or xy is odd.

Case 1: x is odd and y is even

- Let $x = 2m + 1$ (odd) and $y = 2n$ (even), where m, n are integers.

1. **Sum $x + y$:**

$$x + y = (2m + 1) + 2n = 2m + 2n + 1 = 2(m + n) + 1$$

Since $x + y$ is of the form $2k + 1$, it is **odd.**

2. **Product xy:**

$$xy = (2m + 1)(2n) = 2(2mn + n)$$

Since xy is divisible by 2, it is **even.**

Thus, when x is odd and y is even, $x + y$ is **odd.**

Case 2: x is even and y is odd

- Let $x = 2m$ (even) and $y = 2n + 1$ (odd), where m, n are integers.

1. **Sum $x + y$:**

$$x + y = 2m + (2n + 1) = 2(m + n) + 1$$

Since $x + y$ is of the form $2k + 1$, it is **odd.**

2. **Product xy:**

$$xy = (2m)(2n + 1) = 2(2mn + m)$$

Since xy is divisible by 2, it is **even.**

$\downarrow$

Thus, when x is even and y is odd, $x + y$ is **odd.**

Case 3: Both x and y are odd

- Let $x = 2m + 1$ (odd) and $y = 2n + 1$ (odd), where m, n are integers.

1. **Sum $x + y$:**

$$x + y = (2m + 1) + (2n + 1) = 2m + 2n + 2 = 2(m + n + 1)$$

   Since $x + y$ is of the form $2k$, it is **even**.

2. **Product xy:**

$$xy = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1$$

   Since xy is of the form $2k + 1$, it is **odd**.

Thus, when both x and y are odd, xy is **odd**.

Conclusion of Contrapositive

We have shown that:

- If at least one of x or y is odd, then either $x + y$ or xy is odd.

This proves the contrapositive of the original statement. Therefore, the original statement is true:
"If both xy and $x + y$ are even, then both x and y must be even."

6 a) UNIT-III

$a_n = 7a_{n-1} - 10a_{n-2}$ with $a_0 = 2$ and $a_1 = 3$

Sol: Given,

$$a_n = 7a_{n-1} - 10a_{n-2}$$

$$\Rightarrow a_n - 7a_{n-1} + 10a_{n-2} = 0 \longrightarrow (1)$$

It is a II order homogeneous recurrence relation.

Characteristic equation is,

$$r^2 - 7r + 10 = 0$$

$$(r-2)(r-5) = 0$$

$$\boxed{r = 2, 5}$$

Characteristic roots are real and distinct

General solution is,

$$a_n = \alpha_1(2)^n + \alpha_2(5)^n \longrightarrow (2)$$

Put $n=0$ in (2)

$$(2) \Rightarrow \alpha_1(2)^0 + \alpha_2(5)^0 = a_0$$

Given, $a_0 = 2 \Rightarrow \alpha_1 + \alpha_2 = 2$

$$\alpha_2 = 2 - \alpha_1$$

Put $n=1$ in (2)

$$(2) \Rightarrow \alpha_1(2)^1 + \alpha_2(5)^1 = a_1$$

Given, $a_1 = 3 \Rightarrow 2\alpha_1 + 5\alpha_2 = 3$

$$2\alpha_1 + 5(2-\alpha_1) = 3$$

$$2\alpha_1 + 10 - 5\alpha_1 = 3$$

$$-3\alpha_1 = -7$$

$$\boxed{\alpha_1 = \frac{7}{3}}$$

$$\alpha_2 = 2 - \alpha_1 = 2 - \frac{7}{3} = \frac{6-7}{3}$$

$$\boxed{\alpha_2 = \frac{-1}{3}} \text{ in } (2)$$

$$(2) \Rightarrow \boxed{a_n = \frac{7}{3}(2)^n - \frac{1}{3}(5)^n}$$

6 b) Solve the recurrence relation of fibonacci sequence of numbers $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$ given that $f_0 = 0$, $f_1 = 1$

Sol: Given,

$$f_{n+2} = f_{n+1} + f_n$$

$$\Rightarrow \quad f_{n+2} - f_{n+1} - f_n = 0 \longrightarrow (1)$$

It is a II order homogeneous recurrence relation

Characteristic equation is:

$$r^2 - r - 1 = 0$$

$$r = \frac{+1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$$

$$r = \frac{1 \pm \sqrt{5}}{2} = \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$$

The characteristic roots are real and distinct

General solution is:

$$f_n = \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^n \longrightarrow (2)$$

Put $n = 0$ in (2)

$$(2) \Rightarrow \quad \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^0 + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^0 = f_0$$

Given $f_0 = 0 \Rightarrow \alpha_1 + \alpha_2 = 0$

$$\alpha_2 = -\alpha_1$$

Put $n = 1$ in (2)

$$(2) \Rightarrow \quad \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^1 + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^1 = f_1$$

Given $f_1 = 1 \Rightarrow \quad \frac{\alpha_1}{2} + \frac{\sqrt{5}}{2}\alpha_1 + \frac{\alpha_2}{2} - \frac{\sqrt{5}}{2}\alpha_2 = 1$

$$\cancel{\frac{\alpha_1}{2}} + \frac{\sqrt{5}}{2}\alpha_1 \cancel{-\frac{\alpha_1}{2}} + \frac{\sqrt{5}}{2}\alpha_1 = 1$$

$$\frac{\cancel{2}\sqrt{5}}{\cancel{2}}\alpha_1 = 1$$

$$\alpha_1 = \frac{+1}{\sqrt{5}} \qquad \alpha_2 = -\alpha_1$$

$$\alpha_2 = \frac{-1}{\sqrt{5}}$$

Put α_1, α_2 in ② $\Rightarrow$ $\boxed{F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$

(OR)

7 a) Solve the following recurrence relation using characteristic roots

$$a_n + 4a_{n-1} + 6a_{n-2} = 0 \text{ and } a_0 = 2,\ a_1 = 7$$

Sol: Given,

$$a_n + 4a_{n-1} + 6a_{n-2} = 0 \longrightarrow ①$$

① is II order homogeneous recurrence relation

Characteristic Equation is,

$$r^2 + 4r + 6 = 0$$

$$r = \frac{-4 \pm \sqrt{16 - 4(1)(6)}}{2(1)} = \frac{-4 \pm \sqrt{16-24}}{2}$$

$$r = \frac{-4 \pm \sqrt{-8}}{2} = \frac{\cancel{2}(-2 \pm \sqrt{2}i)}{\cancel{2}}$$

$$r = -2 \pm \sqrt{2}i$$

The characteristic roots are complex

General solution is

$$a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$$

$-2+\sqrt{2}i$ $\qquad x + iy = r(\cos\theta + i\sin\theta)$

$x = -2$

$y = \sqrt{2}$

$$r = \sqrt{x^2+y^2} = \sqrt{4+2} = \sqrt{6}$$

$$\tan\theta = \frac{\sqrt{2}}{-2} = \cancel{-35.2^\circ}$$

$$\theta = -35$$

$-2-\sqrt{2}i$

$x = -2$

$y = -\sqrt{2}$

$$r = \sqrt{4+2} = \sqrt{6}$$

$$\tan\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 35^\circ$$

$r_1 = -2+\sqrt{2}i = \sqrt{2}\left(\cos(-35^\circ) + i\sin(-35^\circ)\right)$

$r_2 = -2-\sqrt{2}i = \sqrt{2}\left(\cos(35^\circ) + i\sin(35^\circ)\right)$

$$a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$$

$$= \alpha_1\left[\sqrt{2}\left(\cos(-35^\circ)\right) + i\sin(-35^\circ)\right)\right]^n$$

$$+ \alpha_2\left[\sqrt{2}\left(\cos 35^\circ + i\sin 35^\circ\right)\right]^n$$

$$a_n = \cancel{a_n =} \alpha_1\left[(\sqrt{2})^n\left(\cos n.35^\circ - i\sin n.35^\circ\right)\right]$$

$$+ \alpha_2\left[(\sqrt{2})^n\left(\cos n.35^\circ + i\sin.n\,35^\circ\right)\right]$$

Let coeff. of cos be $K_1 = \cancel{\alpha_1+\alpha_2}$ $\alpha_1+\alpha_2$

sin be $K_2 = \cancel{(\alpha_1-\alpha_2)i}$ $(\alpha_1-\alpha_2)i$

$$a_n = (\sqrt{2})^n\left(K_1\left(\cos n.35^\circ\right) + K_2\left(\sin n.35^\circ\right)\right)$$

Given, $a_0 = 2$

$a_1 = -7$

Put $n=0 \Rightarrow$ $2 = (\sqrt{2})^0\left[K_1.\cos.0.35 + K_2\sin.0.35\right]$

$2 = 1\left[K_1(1) + K_2(0)\right]$

$\boxed{K_1 = 2}$

Put $n=1 \Rightarrow$ $-7 = (\sqrt{2})^1\left[K_1.\cos(1\times35)^\circ + K_2.\sin(1\times35)^\circ\right]$

$-7 = \sqrt{2}\left[2(0.81) + K_2(0.57)\right]$

$-7 = \sqrt{2}\left[2(\approx 0) + K_2\left(\frac{1}{2}\right)\right]$

$K_2 . \frac{1}{\sqrt{2}} = \cancel{7}\ 7$

$\boxed{K_2 = \cancel{7}\ \sqrt{2}}$

$$a_n = (\sqrt{2})^n \left[2 \cdot \cos(n \cdot 35^\circ) \overset{-7\sqrt{2}}{\cancel{+3\sqrt{2}}} \cdot \sin(n \cdot 35^\circ) \right]$$

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7 b) Solve $a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0$ for $n \geq 3$

Sol: Given,

$$a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0 \longrightarrow (1)$$

(1) is a III order homogeneous recurrence relation

Characteristic Equation is

$$r^3 - 9r^2 + 26r - 24 = 0$$

Put $r = 2 \Rightarrow 8 - 36 + 52 - 24 = 0$

$$0 = 0$$

2 is one root

$$\begin{array}{c|cccc} & 1 & -9 & 26 & -24 \\ 2 & 0 & 2 & -14 & 24 \\ \hline & 1 & -7 & 12 & \boxed{0} \end{array}$$

$$(r-2)(r^2 - 7r + 12) = 0$$

$$(r-2)(r-3)(r-4) = 0$$

$$\boxed{r = 2, 3, 4}$$

$$r = \frac{7 \pm \sqrt{49 - 4(12)}}{2}$$

$$r = \frac{7 \pm 1}{2}$$

The characteristic roots are real & distinct

General solution is:

$$a_n = \alpha_1 (2)^n + \alpha_2 (3)^n + \alpha_3 (4)^n$$

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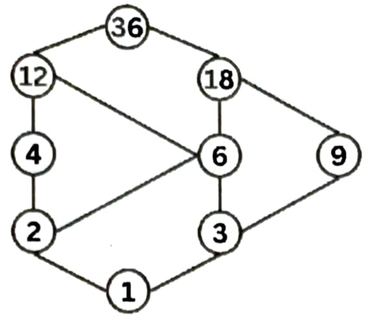
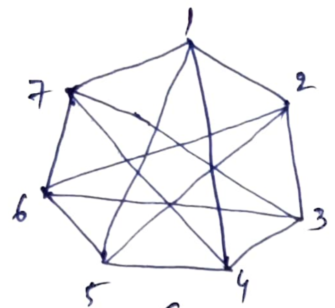
8 (a) Draw the Hasse diagram representing the positive divisors of 36.

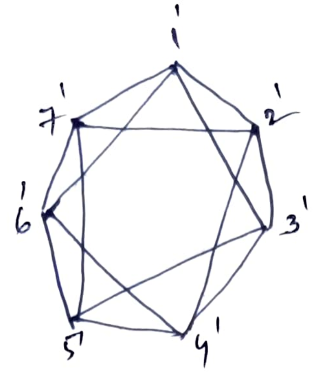
$D_{36} = \{1,2,3,4,6,9,12,18,36\}$



8. b) Show that the following graphs are isomorphic.



G_1



G_2

Ans:

1. Number of vertices in $G_1 = 7$
   Number of vertices in $G_2 = 7$

2. Number of edges in $G_1 = 14$
   Number of edges in $G_2 = 14$

3. Degree Sequence of $G_1 = 4, 4, 4, 4, 4, 4, 4$.
   Degree Sequence of $G_2 = 4, 4, 4, 4, 4, 4, 4$.

4. Adjacency matrices of G_1 & G_2.

G_1

	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

G_2

	1'	2'	3'	4'	5'	6'	7'
1'	0	1	1	0	0	1	1
2'	1	0	1	1	0	0	1
3'	1	1	0	1	1	0	0
4'	0	1	1	0	1	1	0
5'	0	0	1	1	0	1	1
6'	1	0	0	1	1	0	1
7'	1	1	0	0	1	1	0

~~Given~~ Adjacency Matrices are not equal. So we can check using one-to-one correspondence.

One-to-one correspondence:

$f(1) = 1'$, $f(2) = 5'$ $f(3) = 2'$ $f(4) = 6'$ $f(5) = 3'$ $f(6) = 7'$
$f(7) = 4'$

The new adjacency matrices of G_1 & G_2 after row & column transformations;

$$G_1 \qquad \begin{array}{c|ccccccc}
 & 1 & 5 & 2 & 6 & 3 & 7 & 4 \\
\hline
1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
5 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
2 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
6 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
3 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
7 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\
4. & 1 & 1 & 0 & 0 & 1 & 1 & 0
\end{array}$$

$$G_2 \qquad \begin{array}{c|ccccccc}
 & 1' & 2' & 3' & 4' & 5' & 6' & 7' \\
\hline
1' & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
2' & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
3' & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
4' & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
5' & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
6' & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\
7' & 1 & 1 & 0 & 0 & 1 & 1 & 0
\end{array}$$

The adjacency matrices of the given two graphs are same.

So the given 2 graphs are isomorphic.

9 (a) Show that congruence modulo m is an equivalence relation on integers.

We know that the relation "**congruence modulo m**", say R, is defined as

xRy⇔x−y is divisible by m.

For reflexive: Clearly x−x is divisible by m

⇒xRx

So, R is reflexive.

For symmetric:

Let (x,y)∈R

⇒xRy⇒x−y is divisible by m

⇒y−x is divisible by m

⇒yRx

So, R is symmetric.

For transitive:

Let (x,y)∈R and (y,z)∈R

⇒xRy and yRz

⇒x−y=k1m and y−z=k2m

∴x−z=(k1+k2)m

⇒x−z is divisible by m

⇒(x,z)∈R

So, R is transitive.

Hence, R is an equivalence relation.

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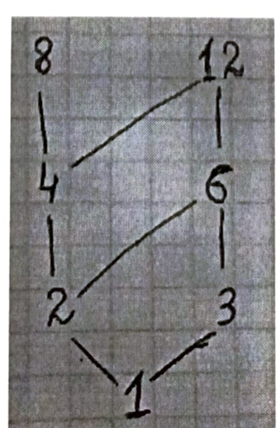
9 (b) Draw the Hasse diagram representing the partial ordering {(a, b)/a divides b}on {1,2,3,4,6,8,12}.

Ans: Partial ordering {(a, b)/a divides b}.on {1,2,3,4,6,8,12}.



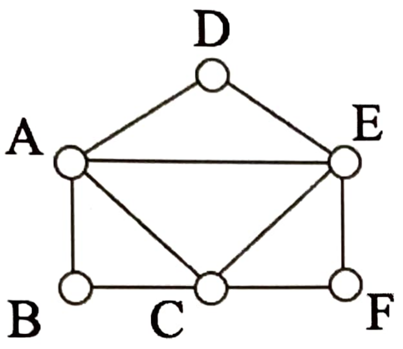
10 (a) Write about Euler's circuit and Hamiltonian cycle with suitable examples.

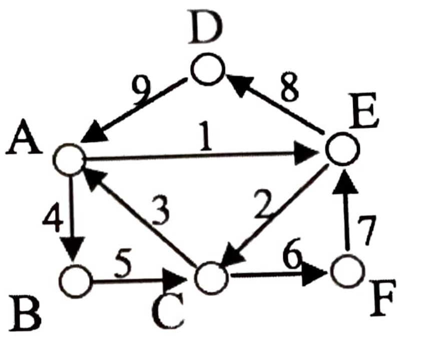
Euler's circuit: An Euler circuit in a graph is a circuit which includes each edge exactly once.

Note: A connected graph has an Euler circuit if and only if every vertex has an even degree.

Examples of Euler Circuit

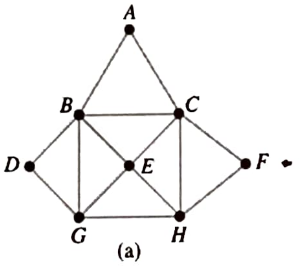
The graph below has several possible Euler circuits. Here's a couple, starting and ending at vertex A: **ADEACEFCBA** and **AECABCFEDA**. The second is shown in arrows.



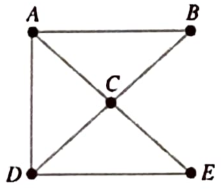


Hamiltonian cycle/circuit: A Hamilton cycle is a cycle in a graph which contains each vertex exactly once.

Example: Hamilton Circuit



(a)



(b)

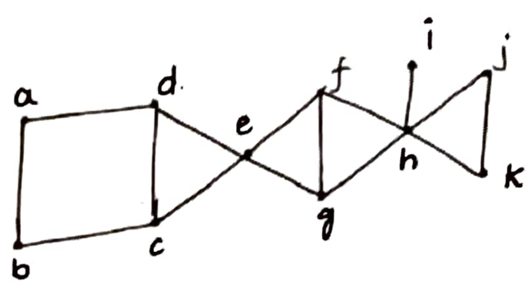
- Graph (a) shown has Hamilton circuit *A, B, D, G, E, H, F, C, A.*
- Graph (b) shown has Hamilton circuit *A, B, C, E, D, A.*

10 (b) Explain DFS algorithm to find spanning tree of a graph with suitable example.

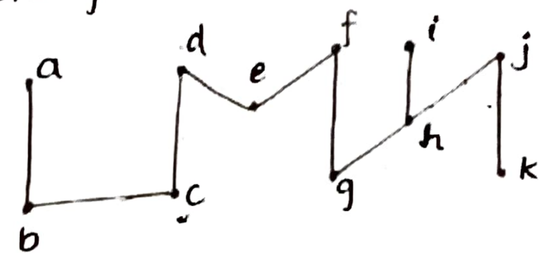
Steps of the DFS Algorithm to Find a Spanning Tree

1. **Start** from an arbitrary vertex v (root of the spanning tree).
2. Mark v as **visited.**
3. Explore all adjacent vertices of v.
   - If an adjacent vertex u has not been visited, include edge (v, u) in the **spanning tree** and perform DFS recursively on u.
4. Continue this process until all vertices are visited.
5. Stop when all vertices have been explored. The edges visited during the traversal form the **spanning tree.**

Example: Graph

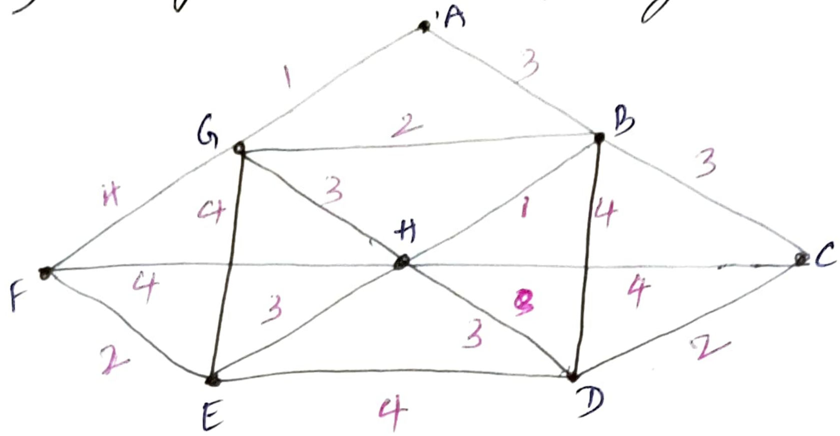


The DFS spanning tree is



(OR)

11. Show step by step kruskals algorithm on the following connected weighted graph and also calculate sum of the weights of the minimal spanning tree?

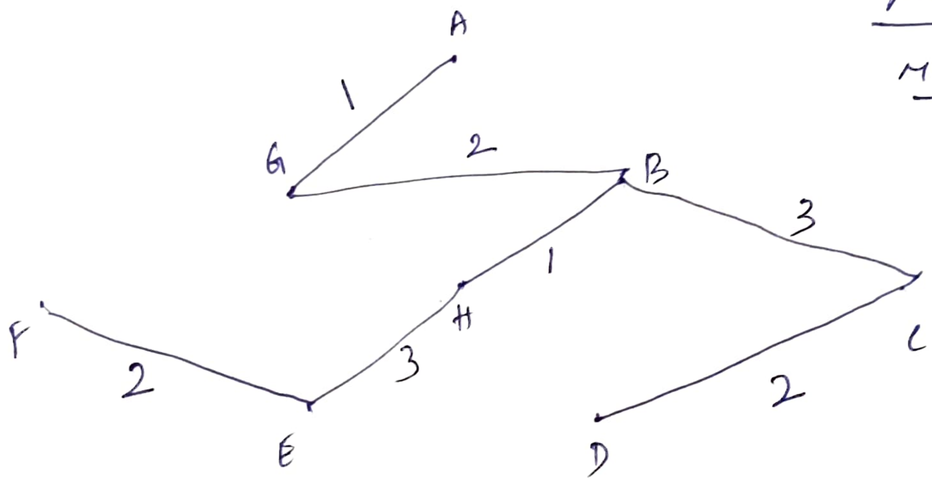


Ans: V : ~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~ ~~F~~

Weight:	1	1	2	2	2	3	3	3	3	3	4	4	4	4	4	4
Edge:	AG	BH	BG	EF	CD	AB	BC	DH	HG	HE	GF	FH	GE	DE	CH	BD.
Select:	yes	yes	yes	yes	yes	no	yes	no	no	yes						

select $n-1$ edges, ∴ 7 edges.

Graph contains various spanning trees but minimum weight is unique.



Total weight is: $1+1+2+2+2+3+3$.

$\boxed{= 14 \text{ units}}$