1) Solve $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$
$a_1 = 7$

Sol: Given,

$$a_n - a_{n-1} - 2a_{n-2} = 0$$

2nd Degree Linear Recc. Relation.

Characteristic Eqn: $r^2 - r - 2 = 0$

$$r = \frac{1 \pm \sqrt{1-4(1)(-2)}}{2}$$

$$r = \frac{1 \pm 3}{2} = 2, -1$$

$$a_n = \alpha_1 (2)^n + \alpha_2 (-1)^n \longrightarrow ①$$

Put $n = 0 \Rightarrow \alpha_1 (2)^0 + \alpha_2 (-1)^0 = a_0$

$\alpha_1 + \alpha_2 = 2 \longrightarrow ⓐ$ (Given $a_0 = 2$)

Put $n = 1 \Rightarrow \alpha_1 (2)^1 + \alpha_2 (-1)^1 = a_1$

$2\alpha_1 - \alpha_2 = 7 \longrightarrow ⓑ$ (Given $a_1 = 7$)

$\alpha_2 = 2\alpha_1 - 7$ in ⓐ

ⓐ $\Rightarrow \alpha_1 + 2\alpha_1 - 7 = 2$

$3\alpha_1 = 9$

$\boxed{\alpha_1 = 3}$ in ⓑ

ⓑ $\Rightarrow 2(3) - \alpha_2 = 7 \Rightarrow \boxed{\alpha_2 = -1}$

Put α_1, α_2 in ① $\Rightarrow a_n = 3(2)^n - (-1)^n$ is req. soln

2.) Solve $a_n = 2a_{n-1} + 3 * 2^n$

Sol: Given, $a_n - 2a_{n-1} = 3 * 2^n \longrightarrow (1)$

1st Order Non-Linear Recc. Relation

General soln is

$$a_n = a_n^{(h)} + a_n^{(P)} \longrightarrow (2)$$

To find $a_n^{(h)}$

$$a_n - 2a_{n-1} = 0$$

Ch. Eqn. $r - 2 = 0$

$$\boxed{r = 2}$$

$$a_n^{(h)} = \alpha (2)^n \longrightarrow (3)$$

To find $a_n^{(P)}$

$$a_n^{(P)} = a_n^{(P_1)} * a_n^{(P_2)}$$

$a_n^{(P_1)} = 3$

Sol: $a_n^{(P_1)} = d$ in (1)

(1) $\Rightarrow$ $d - 2d = 3$

$$\boxed{d = -3}$$

$$a_n^{(P_1)} = -3$$

$a_n^{(P_2)} = 2^n$

$a_n^{(P_2)} = nd2^n$ (2 is a characteristic root)

in (1)

(1) $\Rightarrow$ $nd2^n - 2d(n-1)2^{n-1} = 2^n$

$nd2^n - d(n-1)2^n = 2^n$

$\cancel{2^n}d[n - (n-1)] = \cancel{2^n}$

$$\boxed{d = 1}$$

$$a_n^{(P_2)} = n2^n$$

$$a_n^{(P)} = -3n\,2^n$$

Put values of $a_n^{(H)}$ & $a_n^{(P)}$ in (2)

$$(2) \Rightarrow a_n = \alpha(2^n) - 3n\,2^n$$

$\therefore a_n = 2^n[\alpha - 3n]$ is the required solution.

3) Solve $a_n = 7a_{n-1} - 10a_{n-2}$ with $a_0 = 3$, $a_1 = 5$.

Sol: Given, $a_n - 7a_{n-1} + 10a_{n-2} = 0$

II Order Linear Recurrence relation

Ch. Eqn is $r^2 - 7r + 10 = 0$

$$(r-5)(r-2) = 0$$

$$\boxed{r = 5, 2}$$

The ch. roots are real & distinct. General sol^n is

$$a_n = \alpha_1 5^n + \alpha_2 2^n \longrightarrow (1)$$

Put $n=0$ in (1) $\Rightarrow \alpha_1 5^0 + \alpha_2 5^0 = a_0$

$$\alpha_1 + \alpha_2 = 3 \longrightarrow (a)$$

$$\alpha_2 = 3 - \alpha_1 \text{ in (b)}$$

Put $n=1$ in (1) $\Rightarrow \alpha_1 5^1 + \alpha_2 2^1 = a_1$

$$5\alpha_1 + 2\alpha_2 = 5 \longrightarrow (b)$$

$$5\alpha_1 + 2(3-\alpha_1) = 5$$

$$5\alpha_1 + 6 - 2\alpha_1 = 5$$

$$3\alpha_1 + 6 = 5$$

$$3\alpha_1 = -1$$

$$\boxed{\alpha_1 = \frac{-1}{3}}$$

$$\alpha_2 = 3 + \frac{1}{3}$$

$$\boxed{\alpha_2 = \frac{10}{3}}$$

Put α_1, α_2 in ①

① $\Rightarrow a_n = \frac{-1}{3}(5^n) + \frac{10}{3}(2^n)$ is the required solution.

4) $a_n = a_{n-1} + 3^n$

Sol: Given,

$$a_n - a_{n-1} = 3^n \rightarrow ①$$

I-Order Non-Linear Recc. Relation.

Ch. Eq. is ~~$r-1$~~ General solⁿ is of $a_n = a_n^{(H)} + a_n^{(P)} \rightarrow ②$

To find $a_n^{(H)}$

$$a_n - a_{n-1} = 0$$

Ch. Eq. is $r - 1 = 0$

$$\boxed{r = 1}$$

$$a_n^{(H)} = \alpha(1)^n = \alpha \rightarrow ③$$

To find $a_n^{(P)}$

$a_n^{(P)} = 3^n$ is of the form r^n

Particular solⁿ is $a_n^{(P)} = d\,3^n$ in ①

① $\Rightarrow d\,3^n - d\,3^{n-1} = 3^n$

$$d\,\cancel{3^n}\left[1 - \frac{1}{3}\right] \cancel{3^n}$$

$$d\left[\frac{2}{3}\right] = 1$$

$\boxed{d = \frac{3}{2}}$ $\quad a_n^{(P)} = \frac{3}{2}(3^n) \rightarrow ④$

values of $a_n^{(H)}$ & $a_n^{(P)}$ in ②

from ③, ④ $\Rightarrow$ $a_n = \alpha + \frac{1}{2}(3^{n+1})$

5.) Solve $a_n - 3a_{n-1} + 2a_{n-2} = 0$ for $n \geq 2$

Sol: Given,

$$a_n - 3a_{n-1} + 2a_{n-2} = 0$$

II - Order Linear Recc. Relation

Ch. Eqn. is $r^2 - 3r + 2 = 0$

$$(r-1)(r-2) = 0$$

$$\boxed{r = 1, 2}$$

The characteristic roots are real & distinct

$$\therefore \quad a_n = \alpha_1 1^n + \alpha_2 (2^n)$$

$$a_n = \alpha_1 + \alpha_2 (2^n)$$

6.) Solve $a_n - 6a_{n-1} + 8a_{n-2} = 3^n$ for $n \geq 2$ with $a_0 = 3$, $a_1 = 7$

Sol: Given,

$$a_n - 6a_{n-1} + 8a_{n-2} = 3^n \longrightarrow ①$$

It is a II - Order Non Linear Recurrence Relation

The general sol^n is of the form: $a_n = a_n^{(H)} + a_n^{(P)}$

To find $a_n^{(H)}$

$a_n^{(H)}$ $a_n - 6a_{n-1} + 8a_{n-2} = 0$

Characteristic Equation is $r^2 - 6r + 8 = 0$

$(r-4)(r-2) = 0$

$\boxed{r = 2, 4}$

The ch. roots are real & distinct

$a_n^{(H)} = \alpha_1(2^n) + \alpha_2(4^n) \longrightarrow (2)$

To find $a_n^{(P)}$

$a_n^{(P)} = 3^n$ is of the form $a_n = r^n$

Sol^n is $a_n^{(P)} = d\,3^n$ in (1)

(1) $\Rightarrow$ $d\,3^n - 6d\,3^{n-1} + 8d\,3^{n-2} = 3^n$

$d\,\cancel{3^n}\left[1 - \frac{\overset{2}{\cancel{6}}}{\cancel{3}} + \frac{8}{9}\right] = \cancel{3^n}$

$d\left[\frac{8}{9} - 1\right] = 1$

$d\left[\frac{-1}{9}\right] = 1 \quad \Rightarrow \quad \boxed{d = -9}$

$a_n^{(P)} = -9(3^n) \longrightarrow (3)$

Put (2) & (3) values in a_n

$\Rightarrow a_n = \alpha_1(2^n) + \alpha_2(4^n) - 9(3^n) \longrightarrow (4)$

$\alpha_1(2^n) + \alpha_2(4^n) - 3^{n+2}$

$n=0$ in (4) $\Rightarrow$ $\alpha_1(2^0) + \alpha_2(4^0) - 9(3^0) = a_0$

$$\alpha_1 + \alpha_2 - 9 = 3$$

$$\alpha_2 = 12 - \alpha_1$$

Put $n=1$ in (4) $\Rightarrow$ $\alpha_1(2^1) + \alpha_2(4^1) - 9(3^1) = a_1$

$$2\alpha_1 + 4\alpha_2 - 27 = 7$$

$$2\alpha_1 + 4(12-\alpha_1) = 34$$

$$2\alpha_1 + 48 - 4\alpha_1 = 34$$

$$\cancel{-}\cancel{2}\alpha_1 = \cancel{-}14$$

$$\boxed{\alpha_1 = 7}$$

$$\alpha_2 = 12 - \alpha_1$$
$$= 12 - 7$$
$$\boxed{\alpha_2 = 5}$$

Put α_1, α_2 in (4) $\Rightarrow$ $a_n = 7(2^n) + 5(4^n) - 3^{n+2}$

is the required solution.

7.) Solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 0$ for $n \geq 2$ with $a_0 = 1$, $a_1 = 2$

Sol: Given,

$$a_n - 5a_{n-1} + 6a_{n-2} = 0$$

It is a II Order Homogeneous Recurrence Relation

Characteristic Eq. is $r^2 - 5r + 6 = 0$

$$(r-2)(r-3) = 0$$

$$\boxed{r = 2, 3}$$

The chr. roots are real & distinct

General sol^n is

$$a_n = \alpha_1(2^n) + \alpha_2(3^n) \longrightarrow ①$$

Put $n=0$ in ① $\Rightarrow$ $\alpha_1(2^0) + \alpha_2(3^0) = a_0$

$$\alpha_1 + \alpha_2 = 1$$

$$\alpha_2 = 1 - \alpha_1$$

Put $n=1$ in ① $\Rightarrow$ $\alpha_1(2^1) + \alpha_2(3^1) = a_1$

$$2\alpha_1 + 3\alpha_2 = -2$$

$$2\alpha_1 + 3(1-\alpha_1) = -2$$

$$2\alpha_1 + 3 - 3\alpha_1 = -2$$

$$+\boxed{\alpha_1 = 5}$$

$$\alpha_2 = 1 - \alpha_1$$

$$= 1 - 5$$

$$\boxed{\alpha_2 = -4}$$

Put the values of α_1, α_2 in ①

$\Rightarrow$ $a_n = 5(2^n) - 4(3^n)$ is the required solution.

8.) Solve $a_n - 6a_{n-1} + 8a_{n-2} = n4^n$ for $n \geq 2$

Sol: Given,

$$a_n - 6a_{n-1} + 8a_{n-2} = n4^n \longrightarrow ①$$

It is II - Order Non Linear Recc. Relation.

The general sol^n is $a_n = a_n^{(H)} + a_n^{(P)} \longrightarrow ②$

$\underline{\text{d } a_n^{(H)}}$

$$a_n - 6a_{n-1} + 8a_{n-2} = 0$$

Characteristic Equation is $r^2 - 6r + 8 = 0$

$$(r-4)(r-2) = 0$$

$$\boxed{r = 2, 4}$$

The characteristic roots are real & distinct

$$a_n^{(H)} = \alpha_1(2^n) + \alpha_2(4^n) \longrightarrow (3)$$

<u>To find $a_n^{(P)}$</u>

$$a_n^{(P)} = a_n^{(P_1)} * a_n^{(P_2)}$$

$$a_n^{(P_1)} = n$$

It is linear polynomial

$$a_n^{(P_1)} = d_0 + d_1 n \text{ in } (1)$$

$$(1) \Rightarrow \quad d_0 + d_1 n - 6\left[d_0 + d_1(n-1)\right] + 8\left[d_0 + d_1(n-2)\right] = n$$

$$d_0 + d_1 n - 6d_0 - 6nd_1 + 6d_1 + 8d_0 + 8nd_1 - 16d_1 = n$$

$$3nd_1 - 10d_1 + 3d_0 = n$$

Equating coeff. of $n \Rightarrow$ $3d_1 = 1$

$$\boxed{d_1 = \frac{1}{3}}$$

Constants $\Rightarrow$ $3d_0 - 10d_1 = 0$

$$3d_0 - 10\left(\frac{1}{3}\right) = 0$$

$$\boxed{d_0 = \frac{10}{9}}$$

$$a_n^{(P_1)} = \frac{10}{9} + \frac{1}{3}n$$

$a_n^{(P_2)} = 4^n$ which is a characteristic root

$\Rightarrow a_n^{(P_2)} = dn4^n$ in ①

$$dn4^n - 6d(n-1)4^{n-1} + 8d(n-2)4^{n-2} = 4^n$$

$$d\cancel{4^n}\left[n - \overset{3}{\cancel{6}}(n-1)\cdot\frac{1}{\cancel{4}_2} + 8(n-2)\frac{1}{16}\right] = \cancel{4^n}$$

$$d\left[n - \frac{3}{2}(n-1) + \frac{1}{2}(n-2)\right] = 1$$

$$d[\cancel{2n} - \cancel{3n} - 3 + \cancel{n} - 2] = 2$$

$$d[-5] = 2 \Rightarrow \boxed{d = \frac{-2}{5}}$$

$$a_n^{(P_2)} = \frac{-2n}{5}(4^n)$$

$$a_n^{(P)} = \left(\frac{10}{9} + \frac{n}{3}\right)\left(\frac{-2n}{5}(4^n)\right) \longrightarrow ④$$

Put values of ③ & ④ in ②

② $\Rightarrow a_n = \alpha_1(2^n) + \alpha_2(4^n) + \left(\frac{10}{9} + \frac{n}{3}\right)\left(\frac{-2n}{5}4^n\right)$

is the required solution.

solve $a_{n+1} - 10a_n + 9a_{n-1} = 5 * 9^n$, $n \geq 1$ with $a_0 = 1$, $a_1 = 4$

Sol: Given,

$$a_{n+1} - 10a_n + 9a_{n-1} = 5 * 9^n \rightarrow ①$$

It is a II Order Non-Linear/ ~~Homogeneous~~ Recc. Relation.

The solⁿ is of the form

$$a_n = a_n^{(H)} + a_n^{(P)} \rightarrow ②$$

To find $a_n^{(H)}$:

The associated linear recc. relⁿ is

$$a_{n+1} - 10a_n + 9a_{n-1} = 0$$

The ch. eqn. is

$$r^2 - 10r + 9 = 0$$
$$(r-9)(r-1) = 0$$
$$\boxed{r = 1, 9}$$

The char. roots are real and distinct

$$a_n^{(H)} = \alpha_1 (1)^n + \alpha_2 (9^n)$$
$$a_n^{(H)} = \alpha_1 + \alpha_2 (9^n) \rightarrow ③$$

To find $a_n^{(P)}$:

$$a_n^{(P)} = a_n^{(P_1)} * a_n^{(P_2)}$$

$$a_n^{(P_1)} = 5 \quad \text{(Constant)}$$

Sol is $a_n^{(P_1)} = d$ in ①

① ⇒ $d - 10d + 9d = 5$

$0 = 5$

Now assume $a_n^{(P_1)} = nd$

$① \Rightarrow (n+1)d - 10dn + 9d(n-1) = 5$

$$\cancel{nd} + d - 1\cancel{0}nd + 9\cancel{n}d - 9 = 5$$

$$d - 9 = 5$$

$$\boxed{d = 14}$$

$$a_n^{(P_1)} = 14$$

$a_n^{(P_2)} = 9^n$ which is a characteristic root

Assume $a_n^{(P_2)} = dn9^n$ in ①

$① \Rightarrow d(n+1)9^{n+1} - 10nd9^n + 9(n-1)d9^{n-1} = 9^n$

$$d\cancel{9^n}[9(n+1) - 10n + (n-1)] = \cancel{9^n}$$

$$d[9\cancel{n} + 9 - 1\cancel{0n} + \cancel{n} - 1] = 1$$

$$8d = 1 \Rightarrow \boxed{d = \frac{1}{8}}$$

$$a_n^{(P_2)} = \frac{n}{8}9^n$$

$$a_n^{(P)} = \overset{7}{\cancel{14}} * \frac{n}{\underset{4}{\cancel{8}}}9^n = \frac{7}{4}n9^n \rightarrow ④$$

③, ④ in ②

$\therefore a_n = \alpha_1 + \alpha_2(9^n) + \frac{7}{4}n9^n \rightarrow ⑤$

$1 = 0$ in ⑤ $\Rightarrow \alpha_1 + \alpha_2(9^0) + \frac{7}{4} \times 0 \times 9^n = a_0$

$$\alpha_1 + \alpha_2 = 1 \Rightarrow \alpha_2 = (1 - \alpha_1)$$

$n = 1$ in ⑤ $\Rightarrow \alpha_1 + 9\alpha_2 + \frac{7}{4}(9) = a_1$

$$\alpha_1 + 9\alpha_2 + \frac{63}{4} = 4$$

$$\alpha_1 + 9 - 9\alpha_1 + \frac{63}{4} = 4$$

$$-8\alpha_1 = 4 - \frac{99}{4}$$

$$-8\alpha_1 = \frac{16-99}{4}$$

$$\boxed{\alpha_1 = \frac{83}{32}}$$

$$\alpha_2 = 1 - \frac{83}{32}$$

$$\boxed{\alpha_2 = \frac{-51}{32}}$$

Put α_1, α_2 in (5)

$$\therefore \quad a_n = \frac{83}{32} - \frac{51}{32}(9^n) + \frac{7n}{4} 9^n$$

10.) $a_{n+2} - 6a_{n+1} + 9a_n = 10 * 3^n$, $n \geq 0$

Sol: Given,

$$a_{n+2} - 6a_{n+1} + 9a_n = 10 * 3^n \longrightarrow (1)$$

It is II Order Non-Linear Recc. Relation

The general sol^n is:

$$a_n = a_n^{(H)} + a_n^{(P)}$$

To find $a_n^{(H)}$:

The associated linear recc. relation is

$$a_{n+2} - 6a_{n+1} + 9a_n = 0$$

Ch. Eqn: $r^2 - 6r + 9 = 0$

$$(r-3)(r-3) = 0$$

$$\boxed{r = 3, 3}$$

The characteristic roots are real and equal

$$a_n^{(H)} = (\alpha_1 + n\alpha_2) 3^n \longrightarrow (2)$$

To find $a_n^{(P)}$

$$a_n^{(P)} = a_n^{(P_1)} * a_n^{(P_2)}$$

$$a_n^{(P_1)} = 10$$

R.H.S is constant

assume $a_n^{(P_1)} = d$ in ①

① $\Rightarrow$ $d - 6d + 9d = 10$

$\cancel{10d}$ $4d = 10$

$$\boxed{d = \frac{10}{4} = \frac{5}{2}}$$

$$a_n^{(P_1)} = \frac{5}{2}$$

$$a_n^{(P_2)} = 3^n$$

It is a double characteristic root

Assume $a_n^{(P_2)} = n^2 d\, 3^n$ in ①

① $\Rightarrow$ $(n+2)^2 d\, 3^{n+2}$

$$- 6(n+1)^2 d\, 3^{n+1} + 9n^2 d\, 3^n = 3^n$$

$$(n^2 + 4 + 4n) d\, 3^{n+2} - 6(n^2 + 1 + 2n) d\, 3^{n+1} + 9n^2 d\, 3^n = 3^n$$

$$d\, \cancel{3^n}\left[(n^2 + 4 + 4n)9 - 6 \times 3(n^2 + 1 + 2n) + 9n^2\right] = \cancel{3^n}$$

$$d\left[\cancel{9n^2} + 36 + \cancel{36n} - \cancel{18n^2} - 18 - \cancel{36n} + \cancel{9n^2}\right] = 1$$

$$d\,[18] = 1 \quad \Rightarrow \quad \boxed{d = \frac{1}{18}}$$

$$a_n^{(P_2)} = \frac{n^2}{18} 3^n$$

$$a_n^{(P)} = \frac{5n^2}{36} 3^n \longrightarrow ③$$

Put values of ② & ③ in $a_n = a_n^{(H)} + a_n^{(P)}$

$$\therefore a_n = (\alpha_1 + n\alpha_2) 3^n + \frac{5n^2}{36} 3^n$$

$$a_n = 3^n\left[(\alpha_1 + n\alpha_2 + \frac{5n^2}{36}\right]$$ is the required solⁿ.

ii $a_n - 4a_{n-1} + 4a_{n-2} = 0 \quad n \geq 2$ with $a_0 = 5/2$, $a_1 = 8$

sol: Given,

$$a_n - 4a_{n-1} + 4a_{n-2} = 0$$

It is a II Order Linear Recurrence Relation

Characteristic Eqn. is $r^2 - 4r + 4 = 0$

$$(r-2)^2 = 0$$

$$\boxed{r = 2, 2}$$

The ch. roots are real & equal, the general soln is:

$$a_n = (\alpha_1 + n\alpha_2)2^n \rightarrow ①$$

Put $n=0$ in ① $\Rightarrow (\alpha_1 + 0)2^0 = a_0$

$$\boxed{\alpha_1 = \frac{5}{2}}$$ (Given $a_0 = 5/2$)

Put $n=1$ in ① $\Rightarrow (\alpha_1 + \alpha_2)2 = a_1$

$$2\alpha_1 + 2\alpha_2 = 8$$

$$\cancel{2}\left(\frac{5}{\cancel{2}}\right) + 2\alpha_2 = 8$$

$$2\alpha_2 = 3$$

$$\boxed{\alpha_2 = \frac{3}{2}}$$

Put α_1, α_2 in ① $\Rightarrow a_n = \left(\frac{5}{2} + \frac{3n}{2}\right)2^n$

$a_n = (5+3n)2^{n-1}$ is required soln.

12) Solve $a_n - 6a_{n-1} + 8a_{n-2} = 9 \quad n \geq 2$ $\quad a_0 = 10$

$a_1 = 25$

Sol: Given,

$$a_n - 6a_{n-1} + 8a_{n-2} = 9 \longrightarrow (1)$$

It is a II Order Non-linear recurrence relation

General solⁿ is given by $a_n = a_n^{(H)} + a_n^{(P)} \longrightarrow (2)$

To find $a_n^{(H)}$:

The associated linear eqn. is

$$a_n - 6a_{n-1} + 8a_{n-2} = 0$$

characteristic eqn. $r^2 - 6r + 8 = 0$

$$(r-4)(r-2) = 0$$

$$\boxed{r = 2, 4}$$

The characteristic roots are real & distinct

$$a_n = \alpha_1(2^n) + \alpha_2(4^n) \longrightarrow (3)$$

To find $a_n^{(P)}$:

$$a_n^{(P)} = 9$$

R.H.S is constant

assume, $a_n^{(P)} = d$ in (1)

$(1) \Rightarrow d - 6d + 8d = 9$

$$3d = 9$$

$$\boxed{d = 3}$$

$$a_n^{(P)} = 3 \longrightarrow (4)$$

alues of 3 & 4 in 2

$$2 \Rightarrow a_n = \alpha_1(2^n) + \alpha_2(4^n) + 3 \longrightarrow 5$$

Put $n=0$ in 5 $\Rightarrow \alpha_1(2^0) + \alpha_2(4^0) + 3 = a_0$

$$\alpha_1 + \alpha_2 + 3 = 10$$

$$\alpha_1 + \alpha_2 = 7 \quad \Rightarrow \quad \alpha_2 = 7 - \alpha_1$$

Put $n=1$ in 5 $\Rightarrow \alpha_1(2^1) + \alpha_2(4^1) + 3 = a_1$

$$2\alpha_1 + 4\alpha_2 + 3 = 25$$

$$2\alpha_1 + 4(7-\alpha_1) + 3 = 25$$

$$2\alpha_1 + 28 - 4\alpha_1 + 3 = 25$$

$$-2\alpha_1 = 25 - 31$$

$$2\alpha_1 = 6 \qquad \alpha_2 = 7 - 3$$

$$\boxed{\alpha_1 = 3} \qquad \boxed{\alpha_2 = 4}$$

Substitute α_1, α_2 in 5

$$5 \Rightarrow a_n = 3(2^n) + 4(4^n) + 3$$

$$a_n = 3(2^n + 1) + 4^{n+1}$$

is the required solution.

13.) Solve $a_n - 7a_{n-1} + 10a_{n-2} = 7 * 3^n$ $n \geq 2$

Sol: Given,

$$a_n - 7a_{n-1} + 10a_{n-2} = 7 * 3^n \longrightarrow (1)$$

(1) is II Order Non Linear Recc. Rel^n

The general sol^n is of the form: $a_n = a_n^{(H)} + a_n^{(P)} \longrightarrow (2)$

To find $a_n^{(H)}$:

Write associated linear eqn. of (1)

$$\Rightarrow a_n - 7a_{n-1} + 10a_{n-2} = 0$$

Ch. Eqn. $r^2 - 7r + 10 = 0$

$(r-2)(r-5) = 0$

$\boxed{r = 2, 5}$

$$a_n^{(H)} = \alpha_1(2^n) + \alpha_2(5^n) \longrightarrow (3)$$

To find $a_n^{(P)}$ $\qquad a_n^{(P)} = a_n^{(P_1)} * a_n^{(P_2)}$

$a_n^{(P_1)} = 7$

R.H.S is constant

Assume $a_n^{(P_1)} = d$ in (1)

(1) $\Rightarrow d - 7d + 10d = 7$

$4d = 7$

$\boxed{d = \frac{7}{4}}$

$a_n^{(P_1)} = \frac{7}{4}$

$a_n^{(P_2)} = 3^n$

R.H.S is of the form r^n

Assume $a_n^{(P_2)} = d3^n$ in (1)

$d3^n - 7d3^{n-1} + 10d3^{n-2} = 3^n$

$d\cancel{3^n}\left[1 - 7/3 + \frac{10}{9}\right] = \cancel{3^n}$

$d\left[\frac{9 - 21 + 10}{9}\right] = 1$

$d\left[\frac{-2}{9}\right] = 1 \Rightarrow \boxed{d = \frac{-9}{2}}$

$a_n^{(P_2)} = \frac{-9}{2}3^n$

$$a_n^{(P)} = \frac{7}{4}\left(\frac{-9}{2} 3^n\right)$$

$$a_n^{(P)} = \frac{-63}{8} 3^n \longrightarrow (4)$$

Put (3), (4) in (2)

$$a_n = \alpha_1(2^n) + \alpha_2(5^n) - \frac{63}{8}3^n$$ is the required sol^n.

14) Solve $a_n - 7a_{n-1} + 16a_{n-2} - 12a_{n-3} = 0$ for $n \geq 3$ with

$a_0 = 1$, $a_1 = 4$, $a_2 = 8$

Sol: Given, $a_n - 7a_{n-1} + 16a_{n-2} - 12a_{n-3} = 0$

It is III Order linear recc. rel^n.

Ch. Eqn. $r^3 - 7r^2 + 16r - 12 = 0$

$$\boxed{r = 3, 2, 2}$$

$$a_n^{*} = (\alpha_1 + n\alpha_2)2^n + \alpha_3(3^n) \longrightarrow (1)$$

$n = 0$ in (1) $\Rightarrow$ $\alpha_1 + \alpha_3 = 1 \longrightarrow$ (a)

$n = 1$ in (1) $\Rightarrow$ $(\alpha_1 + \alpha_2)2 + \alpha_3(3^1) = a_1$

$2\alpha_1 + 2\alpha_2 + 3\alpha_3 = 4 \longrightarrow$ (b)

$n = 2$ in (1) $\Rightarrow$ $4\alpha_1 + 8\alpha_2 + 9\alpha_3 = 8 \longrightarrow$ (c)

Solving ⓐ, ⓑ, ⓒ

$$\alpha_1 = 5, \quad \alpha_2 = 3, \quad \alpha_3 = -4 \text{ in } ①$$

① $\Rightarrow$ $a_n = (5 + 3n)2^n - 4(3^n)$ is the required solution.