Law of Logic

Normal Forms (UNIT-I). 1·10
(front page)

For any primitive statement p, q and r any tautology T_0 and any contradiction F_0.

Statement	meaning
① $\sim\sim p \Leftrightarrow p$	— Law of double negation
② $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$ $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$	— demorgan's Laws.
③ $p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	— Commutative Laws.
④ $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$ $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	— associative laws.
⑤ $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	— distributive Laws
⑥ $p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	— Idempotent laws
⑦ $p \vee F_0 \Leftrightarrow p$ $p \wedge T_0 \Leftrightarrow p$	— Identity Laws.
⑧ $p \vee \sim p \Leftrightarrow T_0$ $p \wedge \sim p \Leftrightarrow F_0$	— Inverse Laws (or) Complement laws
⑨ $p \vee T_0 \Leftrightarrow T_0$ $p \wedge F_0 \Leftrightarrow F_0$	— domination laws
⑩ $p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	— Absorption Laws.

Statement | Meaning

(11) $p \to q \Leftrightarrow \sim p \vee q$ — Law of Implication.

(12) $\sim(p \to q) \Leftrightarrow p \wedge \sim q$ — "

(13) $p \to q \Leftrightarrow \sim q \to \sim p$ — Law of contrapositive

(14) $p \vee \sim p \Leftrightarrow T$
$p \wedge \sim p \Leftrightarrow F$ } — Complement law

Ex:- P.T $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$.

$\Rightarrow (\sim p) \wedge (\sim(\sim p \wedge q))$ — demorgan law

$\Rightarrow (\sim p) \wedge (\sim\sim p \vee \sim q)$ — [illegible]

$\Rightarrow \sim p \wedge (p \vee \sim q)$ — Law of double negation

$\Rightarrow (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ — distributive

$\Rightarrow F_0 \vee (\sim p \wedge \sim q)$ — By contradiction

$\Rightarrow \sim p \wedge \sim q = R.H.S$ By identity

$\therefore \sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$.

(2) $(p \wedge q) \to (p \vee q)$ Prove this is a tautology.

we know $p \to q \equiv \sim p \vee q$ (Law of Implication)

$\Rightarrow \sim(p \wedge q) \vee (p \vee q)$

$\Rightarrow \sim p \vee \sim q \vee p \vee q$ — demorgans

$\Rightarrow (p \vee \sim p) \vee (q \vee \sim q)$ — Inverse Law

$\Rightarrow T \vee T$

$\Rightarrow T$.

③ $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

<u>Note:</u> $p \leftrightarrow q = \cancel{(p \wedge \sim q) \vee} (q \wedge \sim p)$ $(p\wedge q) \vee (\sim p \wedge \sim q)$ or $(p \to q) \wedge (q \to p)$

$\sim(p \leftrightarrow q) \Rightarrow \sim(p \to q) \vee \sim(q \to p)$

$\Rightarrow \sim(\sim p \vee q) \vee \sim(\sim q \vee p)$

$\Rightarrow (\sim\sim p) \wedge \sim q \vee (\sim\sim q) \wedge \sim p$

$\Rightarrow p \wedge \sim q \vee q \wedge \sim p$

$\Rightarrow (p \wedge \sim p) \vee (q \wedge \sim q)$.

$\Rightarrow$ R.H.S.

④ $p \uparrow (q \uparrow r) \equiv \sim p \vee (q \wedge r)$ By using truth table.

⑤ $(p \uparrow q) \uparrow r = (p \wedge q) \vee \sim r$

⑥ $p \downarrow (q \downarrow r) \equiv \sim p \wedge (q \vee r)$.

⑦ $(p \downarrow q) \downarrow r \equiv (p \vee q) \wedge \sim r$.

$\to (p \uparrow q) \uparrow r = [\sim(p \wedge q) \uparrow r]$

$= \sim(\sim(p \wedge q) \wedge r)$

$= (p \wedge q) \vee \sim r$.

$\to p \downarrow (q \downarrow r) = p \downarrow \sim(q \vee r)$

$= \sim(p \vee \sim(q \vee r))$

$= \sim p \wedge (q \vee r)$

Duality Law:-

Two formulas A and A^* are said to be the duals of each other if either one can be obtained from the other by replacing $\land$ by $\lor$ and $\lor$ by $\land$.

→ The connectives $\land$ and $\lor$ are also called duals of each other.

→ If the formula A contains the special variable T or F then A^*, its dual is obtained by replacing T by F and F by T.

Ex1: write the duals of (i) $(P\land Q)\lor R$

Sol:- $(P\lor Q)\land R$

(ii) $(P\lor Q)\land F$

Sol: $(P\land Q)\lor T$.

(iii) $\sim(P\land Q)\lor(P\land\sim(Q\lor\sim S))$.

Sol: $\sim(P\lor Q)\land(P\lor\sim(Q\land\sim S))$.

~~Ex2: S.T~~ $\cancel{\sim(\sim P\land\sim(Q\lor R))\Leftrightarrow(P\lor(Q\lor R))}$

$\cancel{A^*}$

Ex-1 $\sim(P\land Q)\rightarrow\sim P\lor(\sim P\lor Q))\Leftrightarrow(\sim P\lor Q)$.

$\Leftrightarrow (P\land Q)\lor(\sim P\lor(\sim P\lor Q))$

$\Leftrightarrow (P\land Q)\lor(\sim P\lor Q)$

$\Leftrightarrow (P\lor\sim P)\land(P\lor Q)\land(Q\lor\sim P)\lor(Q\lor Q)$

$\Leftrightarrow Q\lor\sim P$

$\Leftrightarrow \sim P\lor Q$.

Normal forms

Normal forms:

To determine whether a given compound propositions $A(P_1, P_2 -- P_n)$ is a tautology or a contradiction or atleast satisfiable. and whether two given compound propositions $A(P_1, P_2 -- P_n)$ and $B(P_1, P_2 -- P_n)$ are equivalent, we have to construct the truth tables and compare them.

→ But the construction of truth tables may not be practical, when the no: of primary propositions (variables) $P_1, P_2 -- P_n$ increases.

→ A Better method is to reduce A and B to some standard forms called "normal forms". (or) "Canonical form"

And use them for deciding the nature of A & B and for comparing A and B.

There are two types of normal forms.

① DNF (disjunctive normal form) SOP

② CNF (conjunctive normal form) POS

Note:- We shall use the word 'product' in place of conjunction 'sum' in place of disjunction.

① DNF: (disjunction normal form)

→ A ~~sum~~ product of the variables and their negations are called an elementary ~~sum~~ product

i.e A formula which is equivalent to a given formula and which consists of a ~~product~~ sum of elementary ~~products~~ product is called disjunctive normal form. (DNF).

Ex: Elementary sum: $q, \sim q, p\wedge\sim q, \sim p\wedge\sim q$.

Ex: $p\wedge(p\to q)$ write DNF

Sol: $p\wedge(\sim p\vee q)$.

DNF

$\wedge\ \vee$

$*\ +$

$\Rightarrow \underbrace{(p\wedge\sim p)}_{\text{product}} \underbrace{\vee}_{\text{sum}} (p\wedge q)$ — sum of product.

$\therefore$ this is DNF.

$(\sim p\wedge\sim q)\vee(\sim p\wedge q)\vee$

$(\sim q)\vee(q\wedge q)$

(6) $\sim(p\vee q)\leftrightarrow(p\wedge q)$. write DNF.

$(p\to q)\wedge(q\to p)$

$(\sim p\vee q)\wedge(\sim q\vee p)$

$(\sim p\wedge\sim q)\vee q\wedge\sim q\vee \sim p p$

$\boxed{p\leftrightarrow q :- (p\wedge q)\vee(\sim p\wedge\sim q)}$.

Sol:- $[\sim(p\vee q)\wedge(p\wedge q)]\vee[\sim(\sim(p\vee q))\wedge\sim(p\wedge q)]$

$\Rightarrow [\sim p\wedge\sim q\wedge p\wedge q]\vee[(p\vee q)\wedge(\sim p\vee\sim q)]$

$\Rightarrow [\sim p\wedge\sim q\wedge p\wedge q]\vee[[(p\vee q)\wedge\sim p]\vee[(p\vee q)\wedge\sim q]]$

$\Rightarrow (\sim p\wedge\sim q\wedge p\wedge q)\vee[(p\wedge\sim p)\vee(q\wedge\sim p)]\vee$
$[(p\wedge\sim q)\vee(q\wedge\sim q)]$

$\Rightarrow \underbrace{(\sim p\wedge\sim q\wedge p\wedge q)}_{\text{product}}\underbrace{\vee}_{\text{sum}}(p\wedge\sim p)\vee(\sim p\wedge q)\vee(p\wedge\sim q)\vee$
$(q\wedge\sim p)$.

$\therefore$ sum of product.

Ex:- $(\sim p\vee\sim q)\to(p\leftrightarrow\sim q)$ write DNF.

$(p\to q=\sim p\vee q)$

$(p\to\sim q)\wedge(\sim q\to p)$

$\Rightarrow$ ~~$(\sim p$~~

$(\sim p\vee\sim q)\to($~~$(p\wedge\sim q)\vee(\sim p\wedge q)$~~$)$.

$\Leftrightarrow \sim(\sim p \vee \sim q) \vee [(\sim p \vee \sim q) \wedge (q \vee p)]$

$(x+y)\cdot(x'+y')$

$xx' + xy' + yx'$

$yy'.$

$\Leftrightarrow (p \wedge q) \vee \sim(\ldots)$

$\Leftrightarrow (p \wedge q) \vee [(\sim p \wedge q) \vee \underbrace{(\sim p \wedge p)}_{F_0} \vee \underbrace{(\sim q \wedge q)}_{F_0} \vee (\sim q \wedge p)]$

$\Leftrightarrow (p \wedge q) \vee (\sim p \wedge q) \vee F_0 \vee F_0 \vee (p \wedge \sim q)$.

$\Leftrightarrow (p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q)$. Sum of product.

$\therefore$ this is DNF.

Procedure of obtain the DNF or CNF

① If the connectives $\rightarrow$ and $\leftrightarrow$ are present in a given formula, they are replaced by $\wedge$, $\vee$ and $\sim$.

② If the negation is present before a given formula or a part of the given formula, De Morgan's Laws are applied so that negation is brought before the variables only.

③ If necessary, the distributive and idempotent laws are applied.

CNF :- (conjunctive normal form).

A ~~product~~ sum of variables and their negations are called an elementary ~~product~~ sum

i.e

A formula which is equivalent to a given formula and consist of product of elementary sum is called "conjunctive normal form" of the given formula.

Elementary ~~product~~ sum : $p, \sim p, p \vee \sim p, \sim p \vee q, \sim p \vee \sim q$.

Ex:- $p \wedge (p \to q)$ write CNF.

$$p \wedge (p \to q) \Leftrightarrow \underbrace{p}_{\text{product}} \wedge \underbrace{(\sim p \vee q)}_{\text{sum}} \to \text{product of sum.}$$

∴ this is required form.

Ex:- $\sim(p \vee q) \leftrightarrow (p \wedge q)$ write CNF.

$p \leftrightarrow q$

$(p \to q) \wedge (q \to p)$

$$\Leftrightarrow [(\sim(p \vee q) \to (p \wedge q)] \wedge [(p \wedge q) \to \sim(p \vee q)]$$

$p \to q$

$\equiv \sim p \vee q$

$$\Leftrightarrow [(p \vee q) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)]$$

$$\Leftrightarrow [(p \vee q \vee p) \wedge (p \vee q \vee q)] \wedge [(\sim p \vee \sim q) \vee (\sim p \wedge \sim q)]$$

$$\Leftrightarrow (p \vee q \vee p) \wedge (p \vee q \vee q) \wedge (\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q \vee \sim q)$$

$$\Leftrightarrow (p \vee q) \wedge (p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q).$$

$p \vee p : p$

$p \wedge p : p$

product of sum.

$+ \; \vee$

$* \; \wedge$

∴ this is CNF.

Ex: $P \wedge (P \rightarrow q)$ write CNF.

$\Leftrightarrow P \wedge (\sim P \vee q)$.

$\Leftrightarrow (P \vee F) \wedge (\sim P \vee q)$

$P \vee F_0 \Leftrightarrow P$

$q \wedge \sim q = F_0$

$\Leftrightarrow [P \vee (q \wedge \sim q)] \wedge (\sim P \vee q)$

$\Leftrightarrow (P \vee q) \wedge (P \vee \sim q) \wedge (\sim P \vee q)$.

$\therefore$ this is product of sum.

Principal disjunctive normal form (PDNF).

→ For a given formula, an equivalent formula consisting of disjunction of minterms is known as its PDNF.

→ This is also called "sum of products canonical form".
sum of minterms

Ex: let $P_1, P_2 \ldots P_n$ be n statement variables.

the expression $P_1^* \wedge P_2^* \wedge P_3^* \ldots \wedge P_n^*$.

where P_i^* is either P_i or $\sim P_i$ called minterms.

There are 2^n such minterms. for n variables.

Ex:

P	q	r	minterms
0	0	0	$\sim P \wedge \sim q \wedge \sim r$
0	0	1	$\sim P \wedge \sim q \wedge r$
0	1	0	$\sim P \wedge q \wedge \sim r$
0	1	1	$\sim P \wedge q \wedge r$
1	0	0	$P \wedge \sim q \wedge \sim r$
1	0	1	$P \wedge \sim q \wedge r$
1	1	0	$P \wedge q \wedge \sim r$
1	1	1	$P \wedge q \wedge r$.

$F \rightarrow \sim P$

$T \rightarrow P$

Minterms:

Given a no. of variables, the products in which each variable or its negation, but not both, occurs only once are called minterms.

~~Definition~~: Note:- there are 2^n minterms for n variables.

Ex: for the two variables P and Q, the possible 4 minterms are

$P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge Q$ and $\sim P \wedge \sim Q$.

PDNF:- sum of minterms - whose truth value is 'T'.

PCNF: (" maxterms)! - " " "F".

= product of maxterms.

Principal conjunctive normal forms: (PCNF)

For a given formula, an equivalent formula consisting of conjunction of the maxterms only is known as principal conjunctive normal form (PCNF).

let $P_1, P_2, P_3 \ldots P_n$ be n variable statements.

then the expression $P_1 * \vee P_2 * \vee P_3 * \ldots P_n *$

where P_i or $\sim P_i$ is called maxterms.

Note:- There are 2^n maxterms for n variables.

Maxterms:

Given a no. of variables, the sums in which each variable or its negation, but not both, occurs only once is called the maxterms.

Ex: For two variables P and Q, the possible 4 maxterms are

$P \vee Q$, $P \vee \sim Q$, $\sim P \vee Q$, and $\sim P \vee \sim Q$.

For 3 variables possible maxterms are

Ex)

p	q	r	maxterms
0	0	0	$p \vee q \vee r$
0	0	1	$p \vee q \vee \sim r$
0	1	0	$p \vee \sim q \vee r$
0	1	1	$p \vee \sim q \vee \sim r$
1	0	0	$\sim p \vee q \vee r$
1	0	1	$\sim p \vee q \vee \sim r$
1	1	0	$\sim p \vee \sim q \vee r$
1	1	1	$\sim p \vee \sim q \vee \sim r$.

p q

$F \wedge F \wedge F$

F F F $\sim p \wedge \sim q \wedge \sim r$

$\sim p \wedge q \wedge \sim r$.

$\sim p \vee \sim q \vee \sim r$.

Note:- maxterms are simply duals of minterms.

Ex:- By using truth table construct PDNF & PCNF for $p \to q$, $p \vee q$ and $\sim p \wedge q$.

p	q	$p \to q$	$p \vee q$	$\sim p \wedge q$	$\sim p \wedge q$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	T	F

PDNF $\to$ T.

$p \to q \Leftrightarrow (p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$. $(\sim p \vee \sim q)$.

$(p \vee q) \Leftrightarrow (p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge q)$.

$(\sim p \wedge q) \Leftrightarrow (\sim p \wedge q)$.

PCNF $\to$ F

$(p \vee q) \Leftrightarrow (p \vee q)$.

$(p \to q) \Leftrightarrow (\sim p \vee q)$, $(\sim p \wedge q) \Leftrightarrow (\sim p \vee \sim q) \wedge (\sim p \vee q) \wedge (p \vee \sim q)$.

Ex: Write PDNF for $(\sim p \vee q) \Leftrightarrow (\cancel{\sim p \wedge (q \vee \sim q)}) \vee (\cancel{q \wedge (p \vee \sim p)})$.

Sol: $p \vee \sim p \Leftrightarrow T_0$ (Inverse law) $p \wedge \sim p = F$ product ($\wedge$) → conjunction
$q \wedge T_0 \Leftrightarrow q$ (identity law). $q \vee F = q$ sum ($\vee$) → disjunction.

$(\sim p \vee q) \Leftrightarrow (\sim p \wedge (q \vee \sim q)) \vee (q \wedge (p \vee \sim p))$

$\Rightarrow (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (q \wedge p) \vee (q \wedge \sim p)$

distributive law.

$\Rightarrow (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q) \vee (\sim p \wedge q)$

commutative law.

$\Rightarrow (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q)$. $(p \vee p = p)$ idempotent law

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Ex: Write PDNF for $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$.

$[(P \wedge Q) \wedge (R \vee \sim R)] \vee [(\sim P \wedge R) \wedge (Q \vee \sim Q)] \vee (Q \wedge R \wedge (P \vee \sim P))$

$\boxed{\begin{array}{c} P \vee \sim P = T_0 \\ Q \wedge T_0 = Q \end{array}}$

$\Rightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge R \wedge Q) \vee (\sim P \wedge R \wedge \sim Q) \vee$
$(Q \wedge R \wedge P) \vee (Q \wedge R \wedge \sim P)$.

$\boxed{P \vee P = P}$

$\Rightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)$.

∴ this is PDNF

Ex: $P \vee (P \wedge Q) \Leftrightarrow P$.

$P \vee (P \wedge Q) \Rightarrow P \wedge \underbrace{(Q \vee \sim Q)}_{T_0} \vee (P \wedge Q)$

$\Rightarrow (P \wedge Q) \vee (P \wedge \sim Q) \vee (P \wedge Q)$.

$\Rightarrow (P \wedge Q) \vee (P \wedge \sim Q) = L.H.S$ $\boxed{P \vee P = P}$

$P \Leftrightarrow P \wedge (Q \vee \sim Q)$

$\Leftrightarrow (P \wedge Q) \vee (P \wedge \sim Q) . = R.H.S$

$\therefore L.H.S = R.H.S.$

Ex: $P \vee (\sim P \wedge Q) \Leftrightarrow P \vee Q$.

$P \vee (\sim P \wedge Q) \Rightarrow P \wedge (Q \vee \sim Q) \vee (\sim P \wedge Q)$

$\Rightarrow (P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q) . = L.H.S.$

$P \vee Q \Rightarrow (P \wedge (Q \vee \sim Q)) \vee (Q \wedge (P \vee \sim P))$

$\Rightarrow (P \wedge Q) \vee (P \wedge \sim Q) \vee (Q \wedge P) \vee (Q \wedge \sim P)$.

$\Rightarrow (P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q) = R.H.S$

$\therefore L.H.S = R.H.S.$

Ex: $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$ write PDNF.

$P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P)) \Leftrightarrow P \rightarrow ((\sim P \vee Q) \wedge (P \wedge Q))$

$\Leftrightarrow \sim P \vee [(\sim P \vee Q) \wedge (P \wedge Q)]$

$\Leftrightarrow \sim P \vee [\sim P \wedge (P \wedge Q) \vee Q \wedge (P \wedge Q)]$

$\Leftrightarrow \sim P \vee ((\sim P \wedge P) \wedge (\sim P \wedge Q) \vee \underline{[(Q \wedge P) \wedge P \wedge Q]}$

$\Leftrightarrow \sim P \vee [(F_0 \wedge \sim P \wedge Q) \vee \underline{(P \wedge Q)}]$ $P \wedge Q \wedge Q \wedge Q$.

: PDN

$\Leftrightarrow \sim P \vee (((F_0 \wedge P) \wedge Q) \vee (P \wedge Q))$

$\Leftrightarrow \sim P \vee [(F_0 \wedge Q) \vee (P \wedge Q)]$

$\Leftrightarrow \sim P \vee (F_0 \vee (P \wedge Q))$

$\Leftrightarrow \sim P \vee (P \wedge Q)$

$\Leftrightarrow [\sim P \wedge (Q \vee \sim Q)] \vee (P \wedge Q)$

$\Leftrightarrow (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$.

$$\boxed{\begin{array}{l} Q \wedge Q = Q \\ P \wedge \sim P = F_0 \\ P \wedge F_0 = F_0 \\ P \vee F_0 = P \end{array}}$$

$\therefore$ this is required PDNF.

PCNF

Ex: obtain PCNF of the formula 'S' given by $(\sim P \to R) \wedge (Q \leftrightarrow P)$.

$\Leftrightarrow (\sim P \to R) \wedge (Q \leftrightarrow P)$

$\Leftrightarrow (\sim P \to R) \wedge (Q \to P) \wedge (P \to Q)$.

$\Leftrightarrow (P \vee R) \wedge (\sim Q \vee P) \wedge (\sim P \vee Q)$.

$\Leftrightarrow P \vee R \vee \underbrace{(Q \wedge \sim Q)}_{F_0} \wedge (\sim Q \vee P \vee (R \wedge \sim R)) \wedge (\sim P \vee Q \vee (R \wedge \sim R))$

$\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \sim Q) \wedge (\sim Q \vee P \vee R) \wedge (\sim Q \vee P \vee \sim R)$
$\wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$.

$\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R)$
$\wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$.

$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge$
$(\sim P \vee Q \vee \sim R)$.

$\therefore$ this required PCNF.

PDNF is negation of PCNF.

$S = PCNF$

$\sim S = \sim PDNF$.

$\Leftrightarrow \sim[(P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)]$.

$\Leftrightarrow \sim(P \vee Q \vee R) \vee \sim(P \vee \sim Q \vee R) \vee \sim(P \vee \sim Q \vee \sim R) \vee \sim(\sim P \vee Q \vee R) \vee \sim(\sim P \vee Q \vee \sim R)$

$\Leftrightarrow (\sim P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge \sim R) \wedge (\sim P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R)$.

Ex: Obtain the PCNF for $(Q \to P) \wedge (\sim P \wedge Q)$.

$S: (Q \to P) \wedge (\sim P \wedge Q)$

$\Leftrightarrow (\sim Q \vee P) \wedge (\sim P \wedge Q)$

$\Leftrightarrow (\sim Q \vee P) \wedge [[\sim P \vee (Q \wedge \sim Q)] \wedge (Q \vee (P \wedge \sim P))]$

$\Leftrightarrow (\sim Q \vee P) \wedge (\sim P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (Q \vee P) \wedge (Q \vee \sim P)$

$\boxed{P \vee P = P}$

$\Leftrightarrow (P \vee \sim Q) \wedge (\sim P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (P \vee Q)$.

Ex: Obtain the principal disjunctive and conjunctive normal forms of

Sol: $P \to ((P \to Q) \wedge \sim(\sim P \vee \sim Q))$.

$\Leftrightarrow \sim P \vee ((P \to Q) \wedge \sim(\sim P \vee \sim Q))$

$\Leftrightarrow \sim P \vee ((\sim P \vee Q) \wedge (P \wedge Q))$

$\Leftrightarrow [\sim P \wedge (Q \vee \sim Q)] \vee [[\sim P \wedge (Q \vee \sim Q)] \vee (Q \wedge (P \vee \sim P)) \vee (Q \wedge P)]$

$\Leftrightarrow (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (Q \wedge P) \vee (Q \wedge \sim P) \vee (P \wedge Q)$

$\Leftrightarrow (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$. is the PDNF.

$\sim S \Leftrightarrow$ remaining terms in PDNF.

$\Leftrightarrow (P \wedge \sim Q)$.

PCNF $\Leftrightarrow \sim(\sim S) \Leftrightarrow \sim(P \wedge \sim Q)$

$\Leftrightarrow \sim P \vee Q$ is the PCNF.

Ex: obtain the PDNF of $P \wedge (P \to Q)$.

P	Q	$P \to Q$	$P \wedge (P \to Q)$	minterm
T	T	T	T	$P \wedge Q$ (PDNF)
T	F	F	F	$\sim P \vee Q$
F	T	T	F	$P \vee \sim Q$
F	F	T	F	$P \vee Q$

PCNF.

Ex: obtain the PDNF of $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$.
by using truth table.

Sol:- minterms: $P \wedge Q \wedge R$, $P \wedge Q \wedge \sim R$, $\sim P \wedge Q \wedge R$, $\sim P \wedge \sim Q \wedge R$.

PDNF = sum of minterms.

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)$

Ex: obtain the PCNF of $(\sim P \to R) \wedge (Q \leftrightarrow P)$ by using truth table & replacement process.

Sol:- minterm $\Rightarrow P \wedge \sim Q \wedge R$, $\sim P \wedge Q \wedge R$, $P \wedge \sim Q \wedge \sim R$,
$\sim P \wedge Q \wedge \sim R$, $\sim P \wedge \sim Q \wedge \sim R$.

PCNF = (sum of the minterm)'

$= [(P \wedge \sim Q \wedge R) \vee (\sim P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge \sim R)$
$\vee (\sim P \wedge Q \wedge \sim R)]'$

$\Rightarrow (\sim P \vee Q \vee \sim R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee Q \vee R) \wedge$
$(P \vee \sim Q \vee R)$.

Example : Prove the following

① $p \to (q \to p) \Leftrightarrow p \to (p \to q)$.

L.H.S. $p \to (q \to p) \Leftrightarrow p \to (\sim q \vee p)$

$\Leftrightarrow \sim p \vee \sim q \vee p$

$\Leftrightarrow \sim p \vee p \vee \sim q$

$\Leftrightarrow p \vee \sim p \vee \sim q$

$\Leftrightarrow T \vee \sim q \Leftrightarrow T$.

R.H.S $p \to (p \to q) \Leftrightarrow p \to (\sim p \vee q)$.

$\Leftrightarrow \sim p \vee (\sim p \vee q)$.

$\Leftrightarrow \sim p \vee \sim p \vee q$.

$\Leftrightarrow \sim p \vee q$.

p	q	$q \to p$	$p \to (q \to p)$	$p \to q$	$p \to (p \to q)$
F	F	T	T	T	T
F	T	F	T	T	T
T	F	T	T	F	F
T	T	T	T	T	T

not logical equivalent

(2) $p \to (q \vee r) \Leftrightarrow (p \to q) \vee (p \to r)$.

L.H.S $p \to (q \vee r) \Leftrightarrow \sim p \vee (q \vee r)$.

$\Leftrightarrow \sim p \vee q \vee r$.

$\Leftrightarrow$.

R.H.S $(p \to q) \vee (p \to r) \Leftrightarrow (\sim p \vee q) \vee (\sim p \vee r)$.

$\Leftrightarrow \sim p \vee q \vee \sim p \vee r$.

$\Leftrightarrow (\sim p \vee \sim p) \vee q \vee r$.

$\Leftrightarrow \sim p \vee q \vee r$.

L.H.S = R.H.S.

∴ Both are logically equivalent.

(3) $(p \to q) \wedge (r \to q) \Leftrightarrow (p \vee r) \to q$.

L.H.S $(p \to q) \wedge (r \to q) \Leftrightarrow (\sim p \vee \underline{q}) \wedge (\sim r \vee \underline{q})$.

$\Leftrightarrow (\sim p \wedge \sim r) \vee q$

$\Leftrightarrow q \vee (\sim p \wedge \sim r)$.

$\Leftrightarrow q \vee \sim (p \vee r)$.

$\Leftrightarrow \sim (p \vee r) \wedge q$

$\Leftrightarrow (p \vee r) \to q$.

(4) $\sim(p \leftrightarrow q) \Leftrightarrow (p \vee q) \wedge \sim(p \wedge q)$.

L.H.S $\sim(p \leftrightarrow q) \Leftrightarrow \sim[(p \to q) \wedge (q \to p)]$

$\Leftrightarrow \sim((\sim p \vee q) \wedge (\sim q \vee p))$.

$\Leftrightarrow \sim(\sim p \vee q) \vee \sim(\sim q \vee p)$

$\Leftrightarrow (p \wedge \sim q) \vee (q \wedge \sim p)$.

R.H.S $(p \vee q) \wedge \sim(p \wedge q) \Leftrightarrow \underbrace{(p \vee q)}_{a} \wedge \sim(p \wedge q)$ $\;(\sim p \vee \sim q)$.

$\Leftrightarrow ((p \vee q) \wedge \sim p) \vee ((p \vee q) \wedge \sim q)$

$\Leftrightarrow ((\sim p \wedge p) \vee (\sim p \wedge q)) \vee ((p \wedge \sim q) \vee (q \wedge \sim q))$

$\Leftrightarrow F \vee (\sim p \wedge q) \vee ((p \wedge \sim q) \vee F)$

$\Leftrightarrow (\sim p \wedge q) \vee (p \wedge \sim q)$.

Tautological Implications.

1) prove that $(P \to Q) \Rightarrow (\sim Q \to \sim P)$.

2) prove that $\sim Q \wedge (P \to Q) \Rightarrow \sim P$.

3) ST $\sim (P \to Q) \Rightarrow P$.

Show that following implications

1) $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$.

2) $Q \Rightarrow P \to R$.

3) $(P \wedge Q) \Rightarrow P \to Q$.

Show the following implications without constructing the truth tables,

1) $\sim Q \wedge (P \to Q) \Rightarrow \sim P$.

b) $(P \vee Q) \wedge (\sim P) \Rightarrow Q$.

c) $(P \to Q) \Rightarrow P \to (P \wedge Q)$.

d) $(P \to Q) \to Q \Rightarrow P \vee Q$.

e) $((P \vee \sim P) \to Q) \to ((P \vee \sim P) \to R) \Rightarrow (Q \to R)$.

f) $(Q \to (P \wedge \sim P)) \to (R \to (R \wedge \sim P)) \Rightarrow (R \to Q)$

Tautological Implications without constructing truth tables.

1) $\sim Q \wedge (P \to Q) \to \sim P$.

$\sim Q \wedge (\sim P \vee Q) \to \sim P$.

$(\sim Q \wedge \sim P) \vee (\sim Q \wedge Q) \to \sim P$

$\sim [(\sim Q \wedge \sim P) \cancel{\vee} (\sim Q \wedge Q)] \vee \sim P$.

$(P \vee Q) \wedge (\underline{Q \vee \sim Q}) \vee \sim P$.

$(P \vee Q) \wedge \overset{T}{(T \vee \sim P)}$

$(P \vee Q) \wedge T$

$\sim [(\sim Q \wedge \sim P) \vee F] \vee \sim P$

$\sim (\sim Q \wedge \sim P) \vee \sim P$.

$P \vee Q \vee \sim P$.

$T \vee Q = T$.

2) $(P \vee Q) \wedge \sim P \Rightarrow Q$.

$((P \vee Q) \wedge \sim P) \to Q$

$\sim ((P \vee Q) \wedge \sim P) \vee Q$.

$((\sim P \wedge \sim Q) \vee P) \vee Q$.

$((P \vee \sim P) \wedge (P \vee \sim P)) \vee Q$.

$T \wedge T \vee Q$.

$T \vee Q$

$= T$.

③ $(P \to Q) \Rightarrow (P \to (P \wedge Q))$

$(P \to Q) \to (P \to (P \wedge Q))$.

$(\sim P \vee Q) \to (\sim P \vee (P \wedge Q))$.

$(\sim P \vee Q) \to ((\sim P \vee P) \wedge (\sim P \vee Q))$

$\sim(\sim P \vee Q) \vee (T \wedge (\sim P \vee Q))$

$(P \wedge \sim Q) \vee (\sim P \vee Q)$.

$(P \vee \sim P \vee Q) \wedge (\sim Q \vee \sim P \vee Q)$.

$T \wedge (T \vee \sim P)$

$T \wedge T \Rightarrow T$.

④ $(P \to Q) \to Q \Rightarrow (P \vee Q)$.

$((P \to Q) \to Q) \to (P \vee Q)$.

$((\sim P \vee Q) \to Q) \to (P \vee Q)$.

$(\sim(\sim P \vee Q) \vee Q) \vee (P \vee Q)$.

$((P \wedge \sim Q) \vee Q) \vee (P \vee Q)$.

$((P \vee Q) \wedge (\sim Q \vee Q)) \vee (P \vee Q)$.

$((P \vee Q) \wedge T) \vee (P \vee Q)$.

$F \vee (P \vee Q)$.

$\underline{P \vee Q}$.

not totological implies.

5) $((P \vee \sim P) \to Q) \to ((P \vee \sim P) \to R) \Rightarrow Q \to R$.

$(\sim(P \vee \sim P) \vee Q) \to (\sim(P \vee \sim P) \vee R) \to (Q \to R)$.

$\sim((\sim P \wedge P) \vee Q) \vee ((\sim P \wedge P) \vee R) \to (\sim Q \vee R)$.

$((P \vee \sim P) \wedge \sim Q) \vee (F \vee R) \to (\sim Q \vee R)$.

$(F \wedge \sim Q) \vee QR \to (\sim Q \vee R)$.

$\sim Q \vee R \to (\sim Q \vee R)$.

$\sim(\sim Q \vee R) \vee (\sim Q \vee R)$.

T.

6) $(Q \to (P \wedge \sim P)) \to (R \to (P \wedge \sim P)) \Rightarrow (R \to Q)$.

$[(\sim Q \vee (P \wedge \sim P)) \to (\sim R \vee (P \wedge \sim P))] \to (\sim R \vee Q)$.

$[(\sim Q \vee P) \wedge (\sim Q \vee \sim P)] \to [(\sim R \vee P) \wedge (\sim R \vee \sim P)] \to$
$(\sim R \vee Q)$.

$\sim((\sim Q \vee P) \wedge (\sim P \vee \sim Q)) \vee ((P \vee \sim R) \wedge (\sim P \vee \sim R)) \to$
$(\sim R \vee Q)$.

$(\sim P \wedge Q) \vee (P \wedge Q) \vee [(P \vee \sim R) \wedge (\sim P \vee \sim R)] \to (\sim R \vee Q)$

$\sim((\sim P \wedge Q) \vee (P \wedge Q) \vee ((P \vee \sim R) \wedge (\sim P \vee \sim R))) \vee (Q \vee \sim R)$

$((P \vee \sim Q) \wedge (\sim P \vee \sim Q)) \wedge ((\sim P \wedge R) \vee (P \wedge R)) \vee (Q \vee \sim R)$

$(\cancel{(P \vee \sim Q)} \wedge (\sim P)) \vee$

write DNF

① $\sim(p \vee q) \leftrightarrow (p \wedge q)$.

$(\sim(p \vee q) \rightarrow (p \wedge q)) \wedge ((p \wedge q) \rightarrow \sim(p \vee q))$

$((p \vee q) \vee (p \wedge q)) \wedge (\sim(p \wedge q) \vee \sim(p \vee q))$

$(p \vee q \vee p) \wedge (p \vee q \vee q) \wedge ((\sim p \vee \sim q) \vee (\sim p \wedge \sim q))$

$(p \vee q) \wedge (p \vee q) \wedge ((\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q \vee \sim q))$

$(p \vee q) \wedge (\sim p \vee \sim q) \wedge (\sim p \vee \sim q)$.

$(p \vee q) \wedge \underbrace{(\sim p \vee \sim q)}_{Q}$.

$(p \wedge (\sim p \vee \sim q)) \vee (q \wedge (\sim p \vee \sim q))$.

$(p \wedge \sim p) \vee (p \wedge \sim q) \vee (q \wedge \sim p) \vee (q \wedge \sim q)$

Find CNF

① $(\sim p \rightarrow q) \wedge (p \leftrightarrow q)$

② obtain DNF.

$Q \vee (P \wedge R) \wedge \sim((P \vee R) \wedge Q)$.

1) Obtain PDNF of the following formulas.

a) $P \vee (\sim P \wedge \sim Q \wedge R)$

b) $(Q \wedge \sim R \wedge \sim S) \vee (R \wedge S)$.

CNF

① $p \wedge (p \rightarrow q)$ write CNF.

$p \wedge (\sim p \vee q)$ — product of sum.

② $\sim(p \vee q) \longleftrightarrow (p \wedge q)$ write CNF.

$[\sim(p \vee q) \rightarrow (p \wedge q)] \wedge [(p \wedge q) \rightarrow \sim(p \vee q)]$

$[\cancel{\sim}\sim(p \vee q) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)]$

$(p \vee q) \vee (p \wedge q) \wedge [(\sim p \vee \sim q) \vee (\sim p \wedge \sim q)]$

$[(p \vee q \vee p) \wedge (p \vee q \vee q)] \wedge [(\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q \vee \sim q)]$

$(p \vee q) \wedge (p \vee q) \wedge (\sim p \vee \sim q) \wedge (\sim p \vee \sim q)$.

$\Leftrightarrow (p \vee q) \wedge (\sim p \vee \sim q)$

↓ product ↓ sum

∴ This is required form

Ex: write PDNF for $(\sim p \vee q)$

identity laws:

$p \wedge T =$

$(\sim p \wedge T) \vee (q \wedge T)$.

$(\sim p \wedge (q \vee \sim q))] \vee [q \wedge (p \vee \sim p)]$

$[(\sim p \wedge q) \vee (\sim p \wedge \sim q)] \vee [(q \wedge p) \vee (q \wedge \sim p)]$

$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q)$.

PCNF:

$(\sim P \to R) \wedge (Q \leftrightarrow P)$.

$(\sim P \vee R) \wedge [(Q \to P) \wedge (P \to Q)]$

$((\sim P \vee R) \vee F) \wedge (\sim Q \vee P \vee F) \wedge (\sim P \vee Q \vee F)$.

$[\sim P \vee R \vee (Q \wedge \sim Q)] \wedge [\sim Q \vee P \vee (R \wedge \sim R)] \wedge$
$[(\sim P \vee Q) \vee (R \wedge \sim R)]$

$(\sim P \vee R \vee Q) \wedge (\sim P \vee R \vee \sim Q) \wedge (\sim Q \vee P \vee R) \wedge$
$(\sim Q \vee P \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge$
$(\sim P \vee Q \vee \sim R)$.